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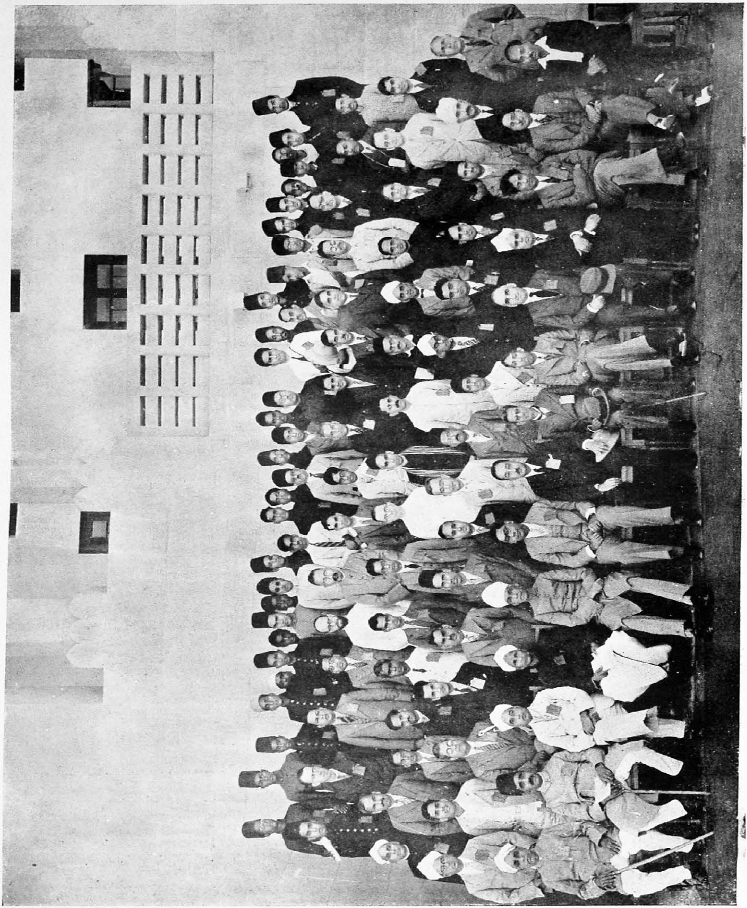

ELEVENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

OSMANIA UNIVERSITY—HYDERABAD (DECCAN)

December 1939.

Sitting (Left to Right)— 1. Prof. M. V. Arunachala Sastry. 2. Prof. Mohd. Abdur Rahman Khan. 3. Prof. A. Narasinga Rao. 4. Dr. R. Vaidyanathaswamy. 5. Mohd. Azhar Hasan, Esq., *Secretary, Home Dept. H. E. H. The Nizam's Govt.* 6. Dr. N. R. Sen. 7. Nawab Mehdi Yar Jung Bahadur, *Education Member and Vice-Chancellor, Osmania University.* 8. Prof. Qazi Mohammad Husain, *Pro-Vice-Chancellor, Osmania University.* 9. Prof. Ram Behari. 10. Prof. F. W. Levi. 11. Dr. T. Vijayaraghavan. 12. Prof. K. S. K. Iyengar. 13. Dr. B. R. Seth. 14. Prof. A. N. Singh.

Standing (1st Row)— 1. Prof. S. Mahadevan. 2. V. Narasimha Murti. 3. Prof. Waheedur Rahman. 4. Dr. M. R. Siddiqi. 5. Prof. Karamchand Dhawan. 6. Prof. Abdul Haq. 7. Prof. A. H. Quraishi. 8. Prof. A. L. Shaik. 9. Prof. K. G. Shah. 10. D. R. Kaprekar, Esq. 11. Prof. V. Rangachariar. 12. Prof. S. A. Hameed. 13. Dr. M. Ziauddin. 14. Prof. S. K. Abhayankar. 15. Prof. K. D. Panday. 16. Prof. M. K. Kevalramani.

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Standing (3rd Row)— 1. Student. 2. Dr. W. F. Kibble. 3. Dr. Qazi Syed Moinuddin. 4. Prof. Mohd. Akram, 5. Father Rafael. 6. Prof. E. Banerji. 7. Prof. K. S. Ramamurti. 8. Prof. K. V. Iyengar. 9. Muneeruddin, Esq. 10. Dr. J. C. Kameshwar Rao. 11. Prof. S. V. Bhagwat. 12. Prof. K. R. Gunjikar. 13. Prof. Kishen Chand. 14. Prof. N. K. Trivedi. 15. Prof. B. B. Bagi. 16. Prof. Mohd. Ali.

Standing (4th Row)— 1 to 11. Students. 12. Prof. T. Venkatarayudu. 13. Student. 14. Student. 15. Mohd. Abdul Aziz, Esq. 16 Student. 17. Prof. V. P. Venkatachari. 18. Student. 19. Prof. Mohd. Khwaja Mohiuddin. 20. Prof. D. D. Kosambi. 21. Student. 22. Syed Wahajuddin Ahmad, Esq. 23—32. Students.

THE MATHEMATICS STUDENT

Volume VII] DECEMBER 1939 [Number 4

OFFICE OF THE 11 MAY MADRAS

The Eleventh Conference

OF THE

INDIAN MATHEMATICAL SOCIETY

The Indian Mathematical Society was invited by the Osmania University, Hyderabad, Deccan, and held its Eleventh Conference on the 21st, 22nd and 23rd of December 1939 in the palatial buildings of the University Arts College.

His Exalted Highness the Nizam was pleased to send a Message of Good Wishes which was read out by the *Hon'ble Nawab Mehdi Yar Jung Bahadur*, Member for Education and Vice-Chancellor, Osmania University, who then declared the Conference open. *Prof. Qazi Mohammad Husain*, Pro-Vice-Chancellor, Osmania University, welcomed the delegates. The Inaugural Address was delivered by *Dr. N. R. Sen*, Ghosh Professor of Applied Mathematics, University of Calcutta.

There were two symposia, one on "Generalised Geometry including Relativity and Field Theories" presided over by *Prof. D. D. Kosombi* of Poona, and the other on "Waring's Problem" under the presidency of *Dr. S. S. Pillai* of Annamalainagar. There were three evening discourses, the first on "Meteoric Astronomy" by *Prof. M. Abdul Rahman Khan*, Hyderabad, the second on "Mathematical Puzzles and Recreations" by *Prof. A. Narasinga Rao* of Annamalainagar and the third on "Mathematics, the Handmaid of Arts, Science and Economics" by *Dr. T. Vijayaraghavan* of Dacca.

As usual there was a Business Meeting and a Discussion on the Teaching of Mathematics in Schools and Colleges. The place of Mathematics in competitive examination in India was one of the subjects which came up for comment.

Nearly 60 papers were contributed to the Conference. In the absence of Dr. R. P. Paranjpye, President of the Society, Prof. N. R. Sen presided at the meetings of the Conference.

The visitors and delegates were treated to an "At Home" on Thursday the 21st December and to a sumptuous dinner on Friday the 22nd December. The Toast of H. E. H. The Nizam was proposed by Prof. Qazi Mohd. Husain. The toast of the guests was proposed by Prof. Kisen Chand and was responded by Prof. N. R. Sen.

The excursions included a visit to the local Observatory, the Golconda Fort and tombs, the Osman Sagar and a 2-day trip to Daulatabad and the rock-hewn caves of Ajanta and Ellora.

The delegates were comfortably lodged in the University Hostel and the catering and other arrangements made by the Reception Committee were highly appreciated by the visitors.

Message from His Exalted Highness the Nizam, to the Eleventh Conference of the Indian Mathematical Society

I am glad to have this opportunity of sending a message of good wishes on the occasion of the Eleventh Session of the Indian Mathematical Conference which is being held in the Capital of my Dominions. I hope you will find your stay here pleasant and that your deliberations will be conducive to the advancement of your great and noble science.

Mathematics is one of the most ancient and highly developed branches of knowledge, and it is a matter of pride to all of us that both the Hindus and the Muslims have contributed a great deal to its early development. I am sure that in the midst of your labours you will find time to see something of this historic City, and that you will realize that our life and culture are just as much a common heritage of the Hindu and Muslim peoples as the Science of which you are the devotees.

I wish all success to your deliberations and hope that your concerted efforts will produce results calculated to bring about a far-reaching advancement of mathematical knowledge.

21st December, 1939.

Reception Committee

Chairman

Prof. Qazi Mohammad Husain,
Pro-Vice-Chancellor, Osmania University.

Secretary

Dr. M. Raziuddin Siddiqi, Professor of Mathematics, Osmania University.

Joint-Secretary

M. Khaja Mohiuddin Esq.,
Lecturer in Mathematics, Osmania University.

Members of the Reception Committee

Nawab Mehdi Yar Jung Bahadur.
Nawab Sir Ameen Jung Bahadur.
Mohd. Abdur Rahman Khan, Esq.
Ahmad Mirza, Esq.
Mohammad Ahmad, Esq.
Mohammad Akram, Esq.
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J. C. Kameshwar Rao, Esq.
Khan Fazal Mohammad Khan, Esq.
Khatib Mahmud Husain, Esq.
Mohd. Khwaja Mohiuddin, Esq.
Kishan Chand, Esq.
R. Krishnamurty, Esq.
Kazi Syed Moinuddin, Esq.
Mohd. Muneeruddin, Esq.
Muzaffaruddin, Quraishi, Esq.

JL

Mohammad Nazeeruddin, Esq.
V. N. Patwari, Esq.
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Shaik Barkat Ali, Esq.
Syed Mohammad Younus, Esq.
Qazi Mohammad Husain, Esq.
Waheedur Rahman, Esq.
V. P. Venkatachari, Esq.
Raziuddin Siddiqi, Esq.

Daily Programme

Thursday, 21st December, 1939

10-30 a.m.	Reading of the Message from His Exalted Highness the Nizam of Hyderabad.
	Welcome Address by the Chairman of the Reception Committee, Prof. Qazi Mohammad Husain, Pro-Vice-Chancellor, Osmania University.
	Inauguration of the Conference by the Hon'ble Nawab Mehdi Yar Jung Bahadur, Member for Education and Vice-Chancellor, Osmania University.
	Report of the Indian Mathematical Society by Dr. Ram Behari, Secretary of the Society.
	Inaugural Address by Prof. N. R. Sen, Ghosh Professor of Applied Mathematics, University of Calcutta.
	Group Photo: Delegates and Members.
2 p.m.	Symposium on "Generalised Geometry including Relativity and Field Theory."
4-30 p.m.	Reception Committee "At Home."
6-30 p.m.	Popular Lecture on "Meteoric Astronomy" by Prof. M. Abdul Rahman Khan, Hyderabad.

Friday, 22nd December, 1939

9 a.m.	Reading of papers.
2 p m.	Symposium on "Waring's Problem."
3-30 p m.	Sightseeing Tour round the City.
6-30 p.m.	Popular Lecture on "Mathematical Recreations" by Prof. A. Narasinga Rao, Annamalai University.

Saturday, 23rd December, 1939

9 a.m.	Reading of papers.
12 noon.	Business Meeting of the Indian Mathematical Society.
2 p.m.	Discussion on the "Teaching of Mathematics."

6 p.m. Popular Lecture on "Mathematics, the Handmaid of Arts, Science and Economics" by Dr. T. Vijayaraghavan of the Dacca University.

7-30 p.m. Departure of delegates to Aurangabad for Excursion to Ellora and Ajanta Caves.

Delegates

Andhra University—
V. Narasimhamurti, Esq.
K. Sambasiva Rao, Esq.

Annamalai University—
A. Narasinga Rao, Esq.
S. S. Pillai, Esq.
V. Seetharaman, Esq.

Bombay University—
D. D. Kosambi, Esq.
N. M. Shah, Esq.

Calcutta University—
F. W. Levi, Esq.

Dacca University—
N. M. Basu, Esq.
T. Vijayaraghavan, Esq.

Delhi University—
Ram Behari, Esq.
B. R. Seth, Esq.

Gwalior State—
S. K. Abhyankar, Esq.

Hindu University, Benares—
V. V. Narlikar, Esq.
Brij Mohan, Esq.

Madras University—
R. Vaidyanathaswamy, Esq.

Muslim University, Aligarh—
M. Ziauddin, Esq.

Mysore University—
K. S. K. Iyengar, Esq.
B. S. Mahadeva Rao, Esq.
K. V. Iyengar, Esq.

Punjab University—
P. Samuels Lall, Esq.

Travancore University—
R. Srinivasan, Esq.
H. Subramania Iyer, Esq.

Opening Speech

BY

THE HON'BLE NAWAB MEHDI YAR JUNG BAHADUR,
Education Member, H. E. H. The Nizam's Government,
Vice-Chancellor, Osmania University.

DELEGATES, LADIES, AND GENTLEMEN,

On behalf of the Osmania University and the Government of Hyderabad I have great pleasure in extending to you a hearty welcome to this beautiful and historic city, the capital of the Premier State of India, where we are gathered together for the Eleventh session of the Indian Mathematical Conference. I hope your stay will be pleasant and comfortable in every way.

In point of years, our University is one of the youngest in India, but during the brief period of its existence, which does not exceed two decades, it has set itself the difficult task of demonstrating to the whole of India that it is not only possible and practicable but actually better to impart instruction in

the higher branches of knowledge (including Mathematics) through one of the principal Indian languages, in this case the *lingua franca* of India. The fact that the Osmania University degrees are now recognised by the British Universities of Oxford, Cambridge and London as well as by other Universities, and that our Medical College Diplomas are recognized by the Royal College of Surgeons, proves the success achieved by this medium of instruction. Our success has encouraged educationists in other Provinces to adopt the aim of imparting education through one or other of the principal languages of India.

This State is not without some title to historic fame. It has had a great past, and the world-renowned caves of Ellora and the frescoes of Ajanta, the forts of Daulatabad and the tombs of Golconda and a host of other monuments bear testimony to the greatness of our past civilization. The names of those who designed these magnificent monuments may be lost to us, but we cannot shut our eyes to the grandeur of their conceptions. Not only have they immortalised in this way the beauty of our ancient sculpture, architecture and painting, but in the carving of their caves and construction of their buildings, they have also exhibited a marvellous technical knowledge of the science of Geometry. For the caves of Ellora as well as those of Ajanta, and many old buildings like the Charminar, are noted for their fine mathematical proportions which add greatly to their charm and beauty and show the consummate art of the people who made them. It will thus be seen that a mathematical sense of proportion has a close relation to art. Today, in modern Hyderabad, Mathematics has been applied to vast works of irrigation and construction, while the architectural style of the Arts College building in which we are assembled shows that the ancient decorative art of the different peoples who inhabit this State is not quite lost to the present generation.

Mathematics, moreover, is the only science that is exact in the true sense of the word. It teaches us the great moral lesson of truth and exactitude; it is by mathematical principles that the mighty universe is governed which we see all around us, with its celestial bodies which, despite their stupendous size and grandeur, move in their stately orbits with a precision that is mathematical. It is Mathematics alone that reveals to us the mechanical secrets of the universe; and since truth is the essence of beauty, it may justly be said that Mathematics in its best sense is a pursuit or cult of truth and beauty which raises it to the pinnacle of the highest form of poetry.

Mathematics has for these reasons a great cultural value and must be allotted its rightful place in any scheme of education, the more so as its value in the sphere of mental discipline and training is incalculable. The Greeks knew this fact and Plato regarded this science as an indispensable factor in education and prescribed a course of mathematics as a subject of universal study. The famous inscription on the gate of the academy of the mathematician Euclid, the founder of Geometry, which read "Let no one enter who cannot geometrise" shows the importance attached to this science by the ancients. This was also the case in India which had the distinction in ancient times of being one of the most advanced countries in this science, especially in its relation to Astronomy.

On its purely practical side, Mathematics is a science of manifold utility. A knowledge of the subject is essential for the study of Physics and

Engineering, and mathematical methods are being increasingly applied to the elucidation of problems in purely lay subjects such as Economics, Logic and Psychology. There is, therefore, ample justification for calling it the "Queen of Sciences".

At one time it was thought that mathematics could not be made interesting or useful to a layman, and mathematicians of a past generation actually took pride in its supposed lack of utility. This is evident from the familiar story of the Cambridge Don, who said with pride regarding a theorem which he had discovered, that its beauty lay in the fact that it could never be put to any practical use! It is needless to say that such notions are now obsolete and as time goes on we find that even the most abstruse branches of mathematics are being increasingly applied to the natural and sociological sciences.

I hope the members of the Conference will be able to pay a visit to the Osmania Observatory which has been doing some original work in co-operation with other observatories and which possesses one of the largest telescopes in India. It has had the good fortune to discover two new stars.

I am pleased to learn that you are devoting some time to a discussion of the methods of teaching mathematics. In this connection, the educational problem is to make it more attractive and interesting to children as well as to grown-up people. I expressly say this because the problem of adult education which is very much in the forefront at the present day, is just now absorbing the attention of the educational authorities.

While much work has been done more work still lies before you, for the possibilities of mathematics are almost infinite.

We in Hyderabad are deeply conscious of the honour you have done us in choosing this city and University for the present session of your Conference. I may say without making too great a claim that some of the members of the Faculty of Mathematics in this University have made an original contribution to mathematical science, more especially in connection with the theory of Einstein.

Let me, in conclusion, express my own good wishes and those of all of us for the success of the Conference. With your permission, I will now do myself the pleasure and honour of declaring this Conference open.

Address of Welcome

BY

THE CHAIRMAN OF THE RECEPTION COMMITTEE,

PROF. QAZI MOHAMMAD HUSAIN,

Pro-Vice-Chancellor, Osmania University

It is with feelings of utmost pleasure that I accord a most hearty welcome to all the members of the Indian Mathematical Conference, on behalf of the Local Reception Committee, to the historic City of Hyderabad and to the Osmania University. It is a very rare occasion that high mathematical talent of such a nature and magnitude could meet together in one place. Such

a gathering in Hyderabad is a matter of gratitude and happiness for all of us. The results of contacts between high intellectual workers in the field of a subject like Mathematics could be of inestimable value and of far-reaching significance as, apart from the mathematical discussions that will be carried out, the original papers that will be read and popular lectures that will be arranged under the auspices of the Conference here, a further great object will be achieved that mathematical minds pursuing the same lines of work and thought will be brought together from all over India, and lasting contacts will be made for the advancement of mathematical knowledge as the result of their co-operative efforts in their lines of study at the various universities and educational institutions in the country. It is hoped that your stay in this City, however short, will be pleasant to yourselves, and I am sure it will be full of benefit to Hyderabad and also it will be conducive to the making of such lasting contacts which I consider as one of the important aims of such intellectual gatherings.

We are all proud of ancient India's great heritage and achievements in the field of Mathematics. The world owes to India some of the basic ideas in the realm of Mathematics. The invention of the decimal system of notation in Arithmetic could be traced back to India, the foundations of Indeterminate analysis were chiefly laid here, and the vast knowledge of the theory and properties of numbers was generated in this country. India could be regarded as the birthplace of Arithmetic and Algebra which have in later times developed into modern analysis. The history of the subject of Mathematics confirms the conclusion that the Indian mind is highly responsive to deep mathematical thought. Eminent mathematicians like Brahma Gupta, Bhaskaracharya of old, and Srinivasa Ramanujam of modern age, not to mention any of the luminaries of the present company, were born on the Indian soil, under Indian conditions. It is a matter for pride to note that the most prominent ancient mathematician, Bhaskaracharya, was born in a village Bijjal Bida in Bid or Bidar District of the Hyderabad Dominions. New times have set in, and it is gratifying to observe that a keen type of scientist and mathematician is coming into the field, in good numbers. The mathematical teaching, as carried out in most places of higher learning in this country, is of a fairly satisfactory and advanced nature, although a great deal of headway has yet to be made in this direction.

The Indian Mathematical Society founded by Mr. Ramaswamy Iyer in 1907 for the advancement of mathematical study and research in India has done a great deal to advance the cause and study of Mathematics. Its achievements in the promotion of mathematical studies and the higher field of research over a period of about 30 years are really appreciable. Its two quarterly journals, one of them containing valuable original papers in the more advanced aspects of the subject and the other the "Mathematics Student," which meets the needs of young research workers and the teachers of Mathematics in schools and colleges, represent the valuable contributions to Mathematics of leading research workers and mathematicians in this country. It has built up a splendid library of mathematical literature and its most welcome feature is the appreciable number of high mathematical journals subscribed by the Society or received in exchange of its own journals. In order to promote the cause of Mathematics, it seems necessary that corporate organisations like the Mathematical

Conference and the Mathematical Society should be assisted by the individual efforts of all mathematicians in every possible way.

The Osmania University has only a short history. The subject of Mathematics is taught here in Urdu up to the M.A. and M.Sc. standard. The instruction of the subject has been all along in efficient hands, two senior members of the mathematics staff being Wranglers of Cambridge, and another senior member being also a high Honours man from Cambridge, with a Doctorate in Mathematics from the Leipzig University. The standards attained in the subject have been fairly satisfactory. The students of Mathematics from Osmania have gone to leading European and English Universities and have given good account of themselves, achieving a series of first classes in Mathematical, Engineering and other Science studies at such Universities. The Local Secretary, Dr. Raziuddin Siddiqi, is a product of Osmania who received his instruction in Mathematics in this department up to his B.A. degree stage. After that, he took his Honours Tripos in Cambridge in two years, instead of three, and later, on such foundations, he got his Doctorate in Mathematics from the Leipzig University. He is considered as one of the leading research workers in the subject in this country, and has produced several original papers which have been published in the leading journals of European countries and of India. He was awarded the gold medal by the Science Academy of Allahabad for the best research work carried out in Science and Mathematics during the last five years. Students of outside Universities—Benares, Aligarh, Madras, Calcutta and other places—are carrying on their researches under his direction from the Mathematics Department here.

The Nizamiah Observatory at Hyderabad, which is one of the valuable institutions of the Osmania University, has been doing excellent work in the domain of Astronomy under the direction of Mr. T. P. Bhaskaran. It has completed a number of volumes about astrographic work, relating to the various regions of the sky that were allotted by the International Astronomical Association to this Observatory. The whole work has been marked with precision and accuracy which is recognised by the highest authorities in the astronomical field. The work done at the Nizamiah Observatory has received recognition on international basis. Apart from the astrographic work, researches about double stars, occultations by the Moon, have been carried on with a telescope of 15″ aperture at the Observatory. Two more instruments are being added to the equipment of the Observatory, one of them a Spectrograph which will open up the regions of the sky with regard to the surface conditions, composition, classification and structure of stars as obtained from their spectra, and the other Spectro-helioscope which will facilitate the study of the surface conditions of the Sun. The students of Mathematics of this department have the unique advantage of paying frequent instructional visits to the Observatory and seeing the actual working of astronomical instruments in the Observatory.

The effective study of a subject cannot be divorced from the history of development of that subject. It may be suggested that the Indian Mathematical Conference and the Indian Mathematical Society may give lead in the matter and encourage the study of ancient Indian and Arabic literature of Mathematics. The Society could draw up a systematic plan and arrange to have it carried out successfully. The ancient important work could be

translated and published, with notes and commentaries. Such work, of course, could only be undertaken by competent scholars of the subject and of the languages concerned. Along with the creation and advance of the subject of Mathematics, it would be well to take stock of what has already been produced by ancient mathematicians of our country as their contributions to this great subject.

Having been in touch with the actual teaching of the subject to the University classes for some years now, it is my privilege to briefly lay before this Conference for their consideration, some observations and suggestions with regard to the work of Mathematics that is being carried on in our educational systems and universities.

The course of Mathematics for a student in schools and colleges consists of 14 years up to the degree stage—10 years at school and 4 years at the University—and if a student takes M.A. in Mathematics also, he devotes a further two years at the University. It may be asked whether as the result of 10 years work in mathematics, the standards reached at the school finishing stage are adequate enough. There seems a great deal of waste and repetition in the course of Mathematics that is done at schools. It may also be scrutinised if the years at the University, up to the degree stage, are being fully utilised and adequate standards produced as a result of four years' work in mathematics by the mathematics student. The same scrutiny may be continued for the M.A. degree course lasting over two years. It may be suggested that a body like the Indian Mathematical Conference will be doing a constructive piece of work, if they would draw up a syllabus for mathematics for the school, degree stage, and M.A. standard, on which a great deal of thought and collaboration of effort by competent members of this Conference may be available and suitable syllabuses, taking all factors into consideration, may be prepared for reference and guidance for the various educational institutions and universities. It is suggested that at the school stage much of the repetition and waste may be avoided and the student may be brought on to the use of the methods of calculus at the end of the course. At the degree stage, almost the same standards may be worked up as at the B.A. Honours degrees of English Universities. With regard to the M.A. stage, it may be considered that some association with research methods in the subject may form a part of the study of the student who takes the M.A. degree. It is now accepted as a principle in all Arts and Science subjects that M.A. and M.Sc. degrees should include the initiation of the student to preliminary research processes; but in Mathematics there is a reluctance, perhaps for sound reasons, to have it followed up. It seems a pity that standards all along the line may not be worked up in such a manner that a person, after 16 years' study of Mathematics, may not be brought into contact with some research methods with a view to enable him to advance the boundaries of knowledge to some extent if he follows up the study of the subject in life.

In order to bring about higher standards of the subject at the various stages, it is most essential that the teachers of Mathematics should have good grounding and control of the subject-matter that they teach. It is suggested that it should be the studied policy of Educational Departments to associate M.A. and M.Sc. trained people with the instruction of higher classes in schools.

Teachers should be given opportunities to join the universities and acquire the discipline of M.A. and M.Sc. studies, just as they are given facilities for training purposes in Training Colleges. Generally, it is considered that people who have taken their B.A. and B.Sc. degrees have adequate grounding to cope with the teaching of higher classes in schools. Being teachers of Mathematics, we know what a vast difference there is between the equipment of a B.A. and M.A. in Mathematics. Training in educational methods is, no doubt, a useful thing, although an opinion may be expressed that "teachers are born," but it may be stated that if a person has got no thorough foundations and grounding of a subject, a training cannot improve his stock of knowledge and abilities. It appears to me that the only effective way of raising the standards of subjects at schools, including Mathematics, is to associate good class M.As. and M.Scs. who have been trained in educational methods with the higher classes of schools. The experience of people would corroborate the fact as to what raising of standards was effected in a school where such men were placed in charge of the instruction of such classes.

With a view to raise the standard of subjects at the university stage, we realise that the functions of a university on the academic side are two,—instruction and research. With regard to the former, it is essential that a university teacher may make himself fully conversant with the "known" knowledge about the branches of the subject that he teaches, by a wide intensive study which gives him the complete mastery and control over the subject that he teaches. With regard to the second, it may be stated that all members of the teaching staff in a university, should be engaged in some kind of original work and research relating to their lines of study. In the hands of a teacher absorbed in the original research sides of a subject, the study of a subject acquires a freshness, power and inspiration that seldom comes to the teaching of one who simply lectures and offers summaries of books as lessons to students. As a matter of fact, "No person can justify his position as a member of the teaching staff of a University, unless he is engaged in some kind of research himself." It is also suggested that the members of the university teaching staff may acquire working knowledge of German, French and other European languages which will help them with regard to their instruction and research in the subject. It may be the avowed aim of universities to expect such grounding of the subjects from all members who join the staff of universities as teachers of higher classes. It may be added that such expectations are made from the members of the staff in this University.

It would be advisable for universities to co-operate in establishing a system of higher extension lectures in Mathematics which may not be entirely research lectures, but may be something of post-M. A. standard, relating to the most advanced, known sides of Mathematics. Lectures on Quantum Mechanics, Mathematical Physics, Relativity Mathematics have been started during the last two or three years at this University, and intimations were sent to all the universities in case any of their scholars or members of the staff would like to take advantage of them. It is suggested that as a corporate effort, such higher lectures may be arranged at suitable times which could conveniently be availed of by advanced students and members of the staff of other places.

These are the days of rapid progress and new knowledge is being added almost every day. It is impossible for any worker, however brilliant he may be, to master all the branches of the subject. Specialisation on firm and comprehensive foundations has to be encouraged. For this purpose, young students and research workers should have ample opportunities of coming into contact with distinguished mathematicians who are working in similar fields, so that they may get new ideas and outlook in the subjects of their interest. Such an object could be achieved by the periodical exchange of Professors or it could be done by introducing the migration system for advanced students with the object of allowing them to study at different centres as is done at most of the continental universities.

The research side of the subject, takes no notice of the fact that researches are being conducted in Applied or Pure Mathematics. They should be carried on as flashes in the mathematical field occur to the worker. With regard to the known stock of knowledge of the subject, however, it is suggested that a special effort may be made to apply this knowledge to the conditions and things round about us in nature. A special aim, along with others, of the mathematical workers may be to develop the sides of Applied Mathematics, in its applications to a wider field in sciences and other subjects. Aeronautics is a coming-up science and there have been enormous developments in the field of this science during the last decade or so. Before long it may come about, that most of the world communications will be through the air. Railways and steamships will be thrown to a relative place in the background and there may be no need to constantly maintain railway tracks, and establish huge harbours and highways on a large scale. India is a vast continent having special facilities for aeronautic travel. Aerodynamics and the application of Mathematics to aeronautical problems may be studied with great advantage as a part of Mathematics,

Mathematics is an abstract subject and is taught in a purely abstract manner. Perhaps it is the great merit relating to this subject, that mathematicians receive a lifelong training in dealing with the abstract conceptions of things, and have their imaginations developed in an enormous degree relating to the subtle concepts of the physical universe. It will, however, be a help to the mathematical thinker if the "concretes" of Mathematics are presented to him in the class room whenever possible, with a view to relieve him of the strenuous strain of the intense abstract thought. For example, in the teaching of Applied Mathematics, demonstrative work with balls, inclined planes relating to rigid dynamics may be supplied in the class room. With regard to higher surfaces in geometry, concrete models will help a great deal, and in wave motion study, various illustrations may be used. In Astronomy which is taught as a part of the Mathematics course, copious use may be made of black spheres and celestial globes apart from the handling of instruments by students with regard to the determination of astronomical data. It is suggested that suitable laboratories may be built up along with the teaching of mathematical subjects and the concrete aspects of the subject may be imported from the Science of Physics, Engineering, etc., and other illustrative appliances and models may be made in the mathematical workshop and laboratory.

It has been pointed out that the chief feature of Mathematics is that it is an abstract science. It represents a discipline for concentration and regulation

of thought that few other subjects could afford. A student of Mathematics of the lower and higher grades is always solving problems, in fact making discoveries with regard to the phenomenon of life at all moments of his work. A mathematician sees things in flashes, He tries to discover the causes why certain things have happened and he is always trying to build up conclusions from sets of facts that constitute the data of a particular problem. He carries on these processes of thought all his life, with most complicated data, with the result that the simpler facts and relations of life come easy to him. He has to turn his attention to any phase of life and he will see through deeper into it understand things more out of it than perhaps a person who has not had such training. With regard to all great undertakings and organisations which are launched on enormous scales by individuals or nations, there are two essential aspects—the imaginative trained thought that conceives all the detailed aspects of a programme, the other is the power of executing the programme. Mathematics will remain as a most vital discipline with regard to the former and the ability to carry out the programme depends on the personal qualities and character of a man who is placed in charge. It has been made out by various eminent writers that a mathematician understands the world all round better in view of his superior discipline of thought. It may, however, be pointed out that his data is oftentimes meagre and inadequate. It will be advisable for him to go about much, observe much, and increase his experience of life by all possible means.

In the end I may say that this is an assembly of seekers after mathematical truths,—a type of truth that is unperishable through all time and is above all personal and world feelings. It is a type of truth that is nearer perfection. It is my signal privilege today to welcome such seekers of truth to the land of Hyderabad and to the Osmania University. I assure all the delegates of the Mathematical Conference that the heart of Hyderabad has welcomed them here.

Secretary's Report for the period March 1938 to December 1939

BY

DR. RAM BEHARI, *Honorary Secretary.*

MR. PRESIDENT, LADIES AND GENTLEMEN :

1. On behalf of the Managing Committee of the Indian Mathematical Society, it is my pleasant duty to extend to you all a hearty welcome to this Conference. We are profoundly thankful to the members of our Society and to other distinguished mathematicians who have undertaken long and tedious journeys to attend this Eleventh Conference of our Society. We are grateful to the many citizens of this famous city who have responded to our invitation and are gracing this occasion with their presence.

We are specially grateful to you, Sir, for the trouble you have taken, in spite of your multifarious duties to come to inaugurate our deliberations.

2. Our Society was founded by the late V. Ramaswamy Aiyar and a group of enthusiastic associates in 1907, and since then it has been growing steadily in strength and usefulness under the wise guidance of its successive presidents.

During the time that has elapsed since we last met at Lucknow in March 1938, 22 new members have been enlisted including 2 life-members.

We are proud to count as one of these new members the eminent mathematician, Professor N. R. Sen who is to deliver the Inaugural Address at this conference.

I regret to have to record the loss which the Society has sustained in the death of Pandit Hem Raj, M.A., Principal and Professor of Mathematics, Doyal Singh College, Lahore, who was a very old and valued member of our Society and an active member of the Managing Committee for over 13 years.

3. Our Society has three main activities viz., holding the biennial Conferences, publication of the two Journals and maintaining a library of advanced books and journals.

The first conference of the Society was held at Madras in December 1916. Since then we have had conferences at Bombay, Lahore, Poona, Bangalore, Nagpur, Trivandrum, Delhi and Lucknow. We are now holding our Eleventh Conference in this unique centre of learning under the auspices of the Osmania University which has given the much desired lead to the country by imparting instruction in all subjects including mathematics, through the mother tongue, up to the highest standard.

It will serve to show the interest evinced by our members in these periodical conferences, if I mention that in spite of the anxious times through which our country is passing on account of the European War, and the inconvenience that might have been caused by the temporary postponement of the Conference, our members and well-wishers have assembled here in large numbers to make this Conference a success by their presence and mathematical contributions. The number of papers that have been communicated to this Conference is nearly 60 as compared to 40 at the last conference. We have also been able to arrange symposia on modern research topics, and a discussion on the courses, and methods of teaching mathematics with a view to bring about a higher standard of mathematical instruction in the country.

4. The Society continues to publish two journals, each of which appears four times a year. The first, viz. '*The Journal of the Indian Mathematical Society*' is edited by Dr. R. Vaidyanathaswami of the Madras University and publishes only original papers and caters for the needs of workers in the field of advanced mathematics. The other one, under the title, '*The Mathematics Student*' is edited by Professor A. Narasinga Rao of the Annamalai University and serves as a link between higher mathematics and the sphere of collegiate mathematics, and also publishes 'News Items' with the object of keeping the readers in touch with one another and with the leading events in the mathematical world in India and abroad.

Besides the journals, the Society has published this year with the financial aid of the Punjab University Dr. Hansraj Gupta's 'Tables of Partitions' a book

which it is hoped will be of great value to workers in the Theory of Numbers.

5. The Library of the Society continues to be under the charge of Professor R. P. Shintre and Professor D. D. Kosambi, of the Fergusson College, Poona. The Library took over the whole of the exchange department, and consequently the journals sent in exchange were mailed from Poona. But before this new scheme could get well under way, the war broke out, and at present, three separate issues of our publications await the chief Censor's permit to be mailed to non-allied countries. On the other hand most of the journals that come to us from abroad are received even now except Journals published in Germany, Czechoslovakia, Poland and journals like 'Science Progress' in England, the publication of which has been suspended.

The circulation work continues as before, but members are forewarned that the war is likely to interrupt our service to some extent.

6. With regard to the financial position of the Society, every year several members of longstanding are taking advantage of the reduced rate of life composition to become life members, and most of the new members are enlisted on the concessional rate of subscription, with the result that the annual income from ordinary membership fees is gradually dwindling. Most of the income got as subscriptions and grants-in-aid from Universities is spent in the course of the ordinary routine work of the Society, and hardly any surplus is left for increasing the efficiency of our service.

The Managing Committee hopes that with the willing co-operation of our members, the financial position will improve by enlisting new members and institutional subscribers for the journals, and that more Universities, Provincial Governments and Ruling Princes will come to our aid by making handsome annual grants.

7. In conclusion, I take this opportunity of giving expression to the thanks of the Society to the Universities that have rendered financial help in the past by giving annual grants-in-aid, and to the various local Governments, Indian States, Universities and other institutions for the facilities they have granted to their officers to enable them to attend this Conference.

I have also great pleasure in expressing, on behalf of the Managing Committee, our thanks to the Vice-Chancellor of Osmania University for giving us this opportunity of assembling here by his kind invitation, and to His Exalted Highness, the Nizam who has been graciously pleased to send a message of encouragement.

Our special thanks are due to Dr. M. R. Siddiqi, his band of volunteers, and other members of the Local Reception Committee for their warm hospitality and excellent arrangements for the comfort of the guests.

Inaugural Address

BY

PROF. N. R. SEN,
Ghosh Professor of Applied Mathematics, Calcutta University

Mathematical and Physical Science

Before proceeding with my address I must express my sincere thanks to you for the honour you have done me by inviting me to deliver this inaugural address, and also state how deeply I appreciate this kindness. I am conscious of my limitations, as I am interested in subjects which may not appeal to mathematicians; nevertheless I will try my best to discharge faithfully the function which I have been called upon to perform.

A well-known physicist of this country once complained to me saying that Mathematics was leading physicists more and more into absurdities. This coming from one who himself has made no mean contribution in his subject is indeed a serious charge. I shall take this opportunity to discuss the question of the relation of Mathematics to the physical sciences, and examine if the latter have really suffered by contamination from Mathematics.

To the common people Mathematics is a subject inspiring awe and reverence. Usually school children dread it, students in Colleges and Universities avoid it, and the average man of the world regards it as a subject too high and difficult for comprehension, though not infrequently in India at least, he is found to take part in discussions in abstruse philosophy with pleasure and enthusiasm. This reputation of Mathematics has no doubt its origin in the abstract nature of the subject, but this is certainly not the whole truth. The reason for the reverence for Mathematics is the common impression regarding the precision and infallibility of its calculation. A subject creates great confidence if its conclusions are arrived at by mathematical processes. It is the application of Mathematics to physical and engineering sciences with their wonderful achievements contributing so much to the development of civilisation, which is the real secret of its success from the ordinary man's point of view. Mathematics is regarded by him as a scientific apparatus by which it is possible to make predictions. Whether it be the question of the amount of electrical energy which will be available from a falling stream of water, or the strength of the walls of a boiler necessary to withstand the pressure of steam, or a premium table which will neither cause an Insurance Company to collapse, nor put severe burden on the shoulders of the participants, in every case it is this powerful weapon of Mathematics which is put into operation to predict results which do not fail. In short the average man's attitude towards Mathematics is determined by the use of Mathematics for solutions of the practical problems of the Applied Sciences. and to the extent they have given him immense advantages in life, an amount of infallibility is attached to mathematical calculations.

It will probably surprise the average man to be told that mathematicians have quite a different idea about their own business. To them Mathematics is not a Science. The methods of observation, experimentation and induction by

which science gains knowledge and advances are unknown in Mathematics. A mathematical truth differs in its very nature from the truth of a scientific discovery. While Science considers reality to lie only in the external world, whose laws are to be deduced mainly by observation and induction, mathematical truths are independent of such considerations. There is thus a great gap between the attitude of the average man and that of the mathematician towards Mathematics. Here one may raise the question of the relation of mathematical truth to the external world, of how and in what manner the processes of nature find a representation within the framework of mathematical systems. An answer to this question can truly interpret the two different attitudes, as well as clearly define the limitations of such a representation.

In technical sciences involving the use of mathematics, mathematical rules and formulae are abundant. The ordinary computer uses formulae often quite mechanically; the expert works out his own formulae under the conditions of his problem. Barring purely empirical results, there is the general tendency to correlate technician's formulae to well-known mechanical or physical laws. The better a formula is in consonance with such a law, the more is the satisfaction of the technician. In ultimate analysis technician's mathematics turns out to be nothing else than an attempt to represent physical laws by mathematical formulae. The mathematical processes of the technicians are well known to be approximations. Klein calls it "approximation mathematics", as distinguished from the "precision mathematics", of the abstract theory. In essence the two are not incompatible, the former is only a phase of the latter, which is a beautiful ever-expanding edifice in an ideal world, and will survive even if the real world crumbles and passes away. We do not propose to go into a detailed discussion of Kleins classification on this occasion. It is enough for us to notice here that engineer's mathematics is, so to say, a sort of calculating machine the inner wheels of which move according to some known, or as yet unknown laws of nature.

In recent years there has been much discussion on the question of what are laws of Nature, and how knowledge of these laws is derived. The attitude of modern science is to recognise knowledge only when it comes through experience, and deductive and inductive processes of thought. The investigator gets his primary knowledge by observations, experiments and measurements. By this process he knows only isolated facts. One can prepare in this manner a catalogue of facts from observations of natural phenomena; this catalogue will indeed be infinitely long. To the primary man, if we think he had not the power to reason, Nature must have revealed herself in this chaotic aspect. Such a view is repugnant to the aesthetic sense of the civilised man. A spiritual need impels him to look for order and harmony in this chaos. This is the origin of his belief that there are laws in Nature. Then arises the need for studying his catalogue closely, rearranging and grouping the data guided by intuition. There appear correlated strings of facts which combine into forms and principles from which by Induction a law of Nature is derived. The process of this derivation in science must be a slow and cautious one. In strictness the truth of such a law can be admitted only in the environments in which the experiences of the investigator lie. Nevertheless, extrapolation of such laws to totally foreign environments, as a tentative measure, is a common tactic adopted by Science in her

endeavour to extend the bounds of her dominion. Thus the receptacle of what we call physical reality is Nature. The laws are the outcome of an urge in intelligent beings to find an order in Nature. It is not the discovery of this order alone that brings full satisfaction of this urge. The mind of Man works on a more ambitious line, and demands such a knowledge of the structure of the Nature as will enable him to make predictions. There thus arises the necessity for constructing an apparatus by which starting from a set of known facts, a different and as yet unknown set may be predicted. Where is the apparatus which will make this wonderful achievement possible? Human intelligence has fashioned it not out of materials taken from Nature, but from imagination working in a totally ideal plane.

While we cannot conceive of physical reality apart from experience, Mathematics has a realm apart from human life and experience. We may say the "reality" in Mathematics does not lie in the world of our experiences. Here we start from certain conceptions, axioms and postulates which need not have any connection with the facts of the Nature. The essential point is that these axioms and postulates should be free from inner contradiction, and eventually form an independent and complete set. On this basement the entire structure of the mathematical system is raised by deductive methods. All the beautiful and elegant theorems known in different branches of Mathematics have their roots deep down in certain sets of fundamental axioms and postulates. Even the finest achievements of abstract Mathematics are, from this point of view mere "tautological transformations". The only principles that govern these transformations are the principles of deductive Logic. Euclidean Geometry for example, is a strictly logical system of this kind based on some axioms and postulates. Its conclusions are exact, and strictly accurate. Its postulates require that through a given point outside a given straight line there should be only one straight line parallel to the given line. This indeed agrees with our usual experience. Bolyai later showed that one can construct a geometrical system in which more than one such parallel may be drawn through the point. In this system the three angles of a triangle are less than two right angles; still the system is logically exact just like the Euclidean system. Riemann followed with a Geometry in which no parallel can be drawn through the point. Geometry thus builds only logical systems the contents of which may look quite absurd from the point of view of our experiences of the world. Mathematical objects have not necessarily any concrete existence. The mathematician starts with objects A, B, C, they may be stars or sticks for him both are equally good. He is concerned only with the relations among the objects, not in the objects themselves. The constructions in Mathematics are only logical systems which are entirely empty in as much as they do not contain any physical reality. This view indeed has not been shared by all mathematical philosophers, but it appears to be the one which is more and more receiving a general support from the side of mathematicians as well as from that of the scientists.

If we admit this to be the real character of abstract Mathematics, we have to find a nexus between the world of facts, and the world of empty logical systems. The latter containing strings of deductions from a set of primary postulates are admirably suited to supply the fundamental need of a predicting machinery about which we have spoken before. Take for instance a differen-

tial equation. It gives us definite information about the nature of the dependence of one variable on another, when we know something about this dependence in isolated cases. This is an apparatus which is extremely handy to reproduce the sequence of a string of facts which may be considered to possess causal links. Take the dynamical problem of a planet in motion in the gravitational field of the Sun. Our inquiry about the motion ultimately reduces to this: if we know the characteristics of the motion of the planet at any time t, how can we calculate the motion at any subsequent time? If the motion be represented by a differential equation, this will give us information about the motion at the instant $t+dt$ when that at the instant t is known. Such a predicting machinery, if it can reproduce facts in the requisite order is nothing else than what we call a law of Nature. All differential laws represent in this manner either the evolution of a physical system in time following what we call a causal chain, or the connection between the infinitely close elements of the system. This is the essence of all Field Theories. But a differential equation is not identical with a Field-law. Its entire content is the peculiar dependence of a varying magnitude on another (no matter what these magnitudes refer to) independently of whether it happens to embody a physical law or not.

Also the modern atomic theory built on the conceptions of certain discontinuous processes clearly shows the same aspect. In classical mechanics conceptions like positional co-ordinate, momentum, and energy are directly representable by ordinary mathematical quantities which are subject to usual algebraic operations. In the domain of atomic mechanics these variables, in order to give correct representation of facts, must be considered to obey a non-commutative algebra. After a trial of over ten years the facts of the atomic motions and transformations were admirably fitted to the algebra of matrices, and the theory of their transformations. The transformation law is a differential one, so that the modern atomic theory has a complicated structure to which the continuous system of partial differential equations, and the discontinuous system of non-commutative algebra have contributed their shares.

The association of Mathematics with the Physical Sciences thus appears as a process of fit. We have seen Mathematics is concerned with the construction of logical systems of different types. But the possibilities here are infinite. These possibilities indeed may be existent only in ideal worlds which are not built after the models of the world of our experiences. Our knowledge of the facts of the Nature is the direct result of our experiences gained by observation and experimentation. Our attempt to find order in these apparently incoherent masses of facts and discover the Laws of Nature leads us to seek which of the known mathematical systems will not only describe these facts correctly, but by a deductive process of thought inherent in the system predict new results. The choice of the system is only a question of fit and convenience. Science cannot deny the possibility that there may be more than one system which may suit her as regards this representation. The scientist will undoubtedly choose the simplest amongst them. Mathematics supplies a steel-frame of construction into which the scientist puts his materials of facts and raises a fine edifice satisfying his inner aesthetic sense of order and beauty.

In the association of facts with logical systems lies the origin of what is known as approximation mathematics of the Applied Sciences. All observations of man are subject to errors, as all his measuring instruments have limitations. On the other hand the mathematical systems are all strictly accurate in the sense that they are logical deductions from certain postulates. Euclid's geometry is indispensable to the engineer in his constructions, but none of his measurements will *exactly* conform to Euclidean geometry. Again take for example, the well-known principle of Energy which the physicist still believes to be inviolable. In the progress of physical science it has made and is still making invaluable contributions. But its verification is never so exact as the mathematical formulation of the principle requires. The principle can thus be taken to be strictly satisfied only in an ideal world, while in the real world of our experiences such deviations are possible as may be correlated to the errors of observation and measurement. Thus the physicist's or engineer's need is a method of calculation which has bearing only on the measurements of the phenomena of the real world. Anything that does not come within the reach of his measuring instruments does not simply interest him. He has no need of irrational quantities, as his measurements do not go beyond a few places of decimals. Nevertheless all his results correspond to certain perfect specifications of an ideal world which is the creation of Mathematics.

The question of the method of choice of a suitable mathematical system is one to which a satisfactory answer is yet to be given. The latitude of an arbitrary choice cannot naturally appeal to one who has set himself the task of constructing a world of harmony and order. As in abstract Mathematics we can construct axiomatic physical theories applicable only in ideal worlds different from our own. For instance, one can imagine a gravitational system in which the law of force is the inverse cube of the distance, instead of the square. This system will have a dynamics different from ours. One may go even further and imagine a world where apples instead of falling down from trees go upwards, where a magnet repels a piece of iron, or where an atom has positrons moving round a heavy electron. Infinities of such systems are imaginable, in which things happen in contradiction with the experiences of the real world. We may call them worlds of miracles in preference to the term "unreal systems". How is the system in which we live to be distinguished from such "unreal systems", in other words how is the real world to be differentiated from these worlds of miracles? We have to face this question when we proceed to exercise our choice of a system for the representation of the facts of Nature. There has been a general tendency in the different branches of Physical Science to use a definite test for this purpose. It is the so-called Action principle, according to which the natural system is characterised by the property, that in it the integral of a certain invariant function is a minimum. Consider a closed space in which you are looking for a law of Nature. Inside of this you start by assuming the possibility of the existence of any one of the worlds of miracles, or of the real world. A mathematical function called the Action function which will have the same value for all observers in any kind of motion, and of which the construction does not depend on a particular law of Nature, may be built and its integral for the whole space concerned taken into consideration. This space which we shall describe by the

word "Field" will be characterised by certain quantities. In fact with every point of the Field will be associated quantities of this kind known as field variables. This association describes a Field which may correspond to the real world, or a world of miracles. Now a variation of the integral of Action will involve a redistribution of the value of the field-quantities, that is passing in general from one world of miracles to another. In this variation the boundary of the Field must not take part; there the conditions existent in the real world are held fixed. The Action Principle states that of all distributions of the field quantities that of the real world makes the integral minimum or strictly speaking an extremum. If then, according to a well-known mathematical principle you change the field-quantities very slightly, that is, pass from the real world to a world of miracles only slightly different from it, the Action integral, to a first approximation will remain unchanged. This condition implies a sort of coherence in the hitherto amorphous Field, and the law of this coherence is the law of Nature. But though a variation principle of this nature is recognized in most branches of physical science, its real import has not been brought to light. We note it is only here in the whole domain of Science that there is a comparison between the natural system on the one hand, and the entire manifold of non-natural systems or worlds of miracles on the other. The natural system possesses a distinct stationary characteristic. It satisfies our aesthetic taste to think that among the infinity of possible systems there is one which has a unique property, this being the natural system. But this uniqueness is bound up with particular forms of the invariant Action function. Why this should be so is a mystery of Nature which as yet remains unrevealed.

In the domain of atomic physics a different test is applied to restrict the choice of systems. In interpreting atomic phenomena in terms of electronic motions inside the atom, it so happens that these motions can be associated with certain sets of integral and half integral numbers called quantum numbers. A particular association of these quantum numbers determines a state of the atom. Among all possible electronic motions within the atom, only the motion in the natural system satisfies the Exclusion Principle of Pauli, which allows one and only one electron in a completely defined quantum state. One can certainly think of other models of atoms but they do not populate the world of our experience. In the case of discontinuous atomic phenomena the selection of a suitable mathematical system is thus more direct than in the Field theories.

These considerations show how Mathematics and Physical Sciences originate, and how they develop along their own lines. In the progress of Science Mathematics is bound to play a prominent role. The exact laws of Nature are expressed as mathematical equations. The conclusions drawn from these equations are again interpreted in terms of natural phenomena and often lead to new knowledge. Yet physical laws do not serve any mathematical interest nor are mathematical systems primarily designed to represent physical laws. The domain of Mathematics is closed and selfcontained and is independent of physical realities. If it leads you to an unreal world, it is due to your own choice.

ABSTRACTS OF PAPERS

M. Venkatarama Ayyar and M. Bhimasena Rao, Bangalore.

On types of solutions of $x^3+y^3+z^3=1$ in integers.

S. Ramanujan had given this problem as Question 681 in the J. I. M. S. for 1915, adding 6 illustrative examples of which the last was $67402^3-83802^3+65601^3=1$. A partial solution of this was given by Prof. N. B. Mitra in the J. I. M. S. for 1921, verifying only the first two cases and omitting the rest. In the present paper, the authors have brought out the underlying unity among all the examples, which come under type 4, the type being defined by the value of $(1-z)/(x+y)$. The same method is extended to obtain examples of other types and complete solutions for types 3, 4, 7 and 13 are furnished. As simple examples may be mentioned,

under type 3,	$3753^3-2676^3-3230^3=1.$
	$3753^3-5262^3+4528^3=1.$
under type 7,	$94^3-103^3+64^3=1.$
under type 13,	$8343^3-8657^3+4083^3=1.$ etc.

The formula given by Ramanujan in Q. 441 (J. I. M. S. 1913),

$$(6a^2-4ab+4b^2)^3-(4a^2-4ab+6b^2)^3-(3a^2+5ab-5b^2)^3=(5a^2-5ab-3b^2)^3$$

is of our type 4 and is a variation of a formula given by J. R. Young much earlier,

$$(2d^2+4d+42)^3-(2d^2-4d+42)^3+(d^2-16d-21)^3=(d^2+16d-21)^3$$

Similar formulae for types 3, 7, 13, 19 etc. are also obtained.

D. R. Kaprekar, Devlali.

Demlofication of Numbers in Arithmetic Progression.

Take an A. P. say $n, n+d, n+2d, \cdots$ and write the terms one below the other advancing the unit digit one step to the left each

time. When the numbers are added we have a group of recurring digits. Thus for $n=17$, $d=6$ the recurring group is 814. In this paper it is pointed out that the repeating group depends on d and that there are 4 cases.

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               17
              23
             29
            35
           41
          47
         53
        59
       65
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··1 4 8 1 4 8 1 4 7
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Case (*i*) If d is a multiple of 9 the group consists of 1 digit only.

Case (*ii*) If d is a multiple of 3 but not of 9 or 10, the group consists of 3 digits which may be any one of the following or a cyclic permutation of the same

074; 185; 296; 370; 481; 592

Case (*iii*) If d is any number not divisible by 3 or 10 the repeating group consists of 9 digits which may be any cyclic permutation of 6 groups of 9 digits.

Case (*iv*) Covers the case of d being a multiple of 10.

S. S. Pillai

On Waring's Problem

Dickson has proved that $g\ (5) \leqslant 55$ and $g\ (4) \leqslant 35$. In this paper I show that $g\ (5) \leqslant 41$ and $g\ (4) \leqslant 27$.

K. Sambasiva Rao, Andhra University, Waltair.

On Waring's Problem for Fifth Powers.

Let $G(k)$ denote the least integer s such that the Diophantine equation

$$N = x_1^k + \cdot \cdot \cdot + x_s^k,$$

is solvable in positive integers for all sufficiently large integers N.

It was proved by Hardy and Littlewood that $G(5) \leqslant 41$. Subsequently several writers improved it: James to $G(5) \leqslant 35$; Estermann to $G(5) \leqslant 29$; Hua to $G(5) \leqslant 28$.

The purpose of the present paper is to prove that

$$G(5) \leqslant 25.$$

The author has been able to arrive at this result by improving, among other things, a theorem of Davenport on 'admissible exponents'. Other consequences of the method are

$$G(6) \leqslant 40; \quad G(7) \leqslant 56$$

These are improvements on the previous results.

Hansraj Gupta, Hoshiarpur.

Waring's Problem for powers of primes—II.

In a recent paper Dr. S. S. Pillai has examined one of my conjectures and proved the theorem :—If $l \leqslant r \leqslant 2^n - 2l - 2$ and $n \geqslant 20$, then every integer not exceeding N is the sum of at most I nth powers of primes $\geqslant 1$ where

$$N = e^{(1 \cdot 6)^n/6} \cdot 31^n,$$
$$3^n = l \cdot 2^n + r$$
and
$$I = 2^n + l - 2$$

He has not stated how far his theorem holds for values of $n < 20$. The object of the present paper is to study the validity of Pillai's Theorem in this case and strengthen the following conjecture : If $n \geqslant 7$, then every integer can be expressed as the sum of at most I nth powers of primes $\geqslant 1$. For $n < 7$ this conjecture is easily seen to be false.

V. Narasimhamurti, Waltair.

On a Problem in Arrangements.

PROBLEM.—"$2n+1$ people are invited out to dinner on n different days. Is it possible to arrange them round a circular table in such a way that no person has the same neighbour on different days ?"

Suppose the following is a solution,

1st row $(y_{n+1}+n,\ y_n+n,\ \cdots,\ y_2+n,\ y_1,\ y_2\ \cdots,\ y_n,\ y_{n+1})$
2nd row $(y_{n+1}+n+1,\ y_n+n+1,\ \cdots,\ y_2+n+1,\ y_1,\ y_2+1,\ \cdots,$
... ... $y_n+1,\ y_{n+1}+1)$
nth row $(y_{n+1}+n+n-1,\ y_n+n+n-1,\ \cdots,\ y_2+n+n-1,\ y_1,$
$y_2+n-1,\ \cdots,\ y_n+n-1,\ y_{n+1}+n-1)$

where $y_1=1,\ y_2=2,\ y_3=2+x_1,\ y_4=2+x_1+x_2,\ \cdots$
$y_{n+1}=2+x_1+x_2+\ \cdots\ +x_{n-1}$;
and the x's are all among the numbers $1, 2, 3, \cdots, n-2, n-1, n+1, n+2, \cdots, 2n-1$.

It should be noted that any number —(excepting the number $y_1=1$)—which is congruent to r (mod $2n$) is to be replaced by r if $2 \leqslant r \leqslant 2n-1$; and by $2n+1$ or $2n$ if $r=1$ or 0 respectively. We have thus all the $2n+1$ numbers from 1 up to $2n+1$.

Now the solution will be possible if all the x's satisfy the following two conditions,

(1) $x_r + x_s \neq 2n$ for any r and s
(2) $x_m + x_{m+1} + \cdots + x_{m+t}$ is not divisible by n.

For, the second ensures that the numbers in each row are different, and the first excludes the possibility of any two neighbours in one row appearing again as neighbours in another row.

The above conditions are satisfied if we choose $x_1 = 1$, $x_2 = 2n-2$, $x_3 = 3$, $x_4 = 2n-4$, $\cdot\ \cdot\ \cdot\ \cdot$ $x_{n-1} = n-1$ or $n+1$ according as n is even or odd.

S. M. Kerawala, Aligarh.

The Problem of the Play of Thirteen.

The problem is generalised in this paper for three players in the form: Three players A, B, C are given three well-mixed packs, each consisting of the same n different cards. They expose together singly cards out of their respective packs. What is the probability that no two of any set of three cards exposed together be the same? If v_n is the probability, it is shown that v_n satisfies the difference equation

$$v_{n+5} = \frac{(n+4)(n^2+8n+17)}{(n+3)(n+5)^2} v_{(n+4)} + \frac{(n^2+8n+17)}{(n+4)(n+5)^2} v_{(n+3)}$$
$$+ \frac{(n^2+8n+13)}{(n+3)^2(n+4)(n+5)^2} v_{(n+2)} + \frac{2(n^2+5n+3)}{(n+2)(n+3)^2(n+4)(n+5)^2} v_{n+1}$$
$$- \frac{4v_n}{(n+5)^2(n+3)^2(n+2)(n+1)}.$$

B. S. Madhava Rao, Bangalore.

On the limits of the roots of a polynomial equation

A simple proof is given of a theorem due to Laguerre on the limits of the roots of a polynomial equation all of whose roots are real and distinct. Further theorems are proved which sharpen these limits.

K. V. Iyengar, Bangalore.

A deepening of the Binomial inequality.

We know that $E = \dfrac{x^n - 1}{x - 1}$ lies in (nx^{n-1}, n). This inequality is deepened by elementary methods so that depending on the value of the exponent n, $(1 < n < 2$, etc.) we can determine in which of the subintervals formed by marking the A. M. G. M. and H. M. of the extremes of the above interval, E lies. A particular case of the same where n is an integer is given as a problem in Hardy-Littlewood-Polya, *Inequalities.*

F. W. Levi, Calcutta.

Pairs of inverse moduls.

Two submoduls A and A′ of a (commutative) field F will be said to be *inverse* if the elements different from zero of A are inverse to those of A′. If especially A = A′, the modul is *self-inverse.* If the characteristic of F is different from 2, one gets all the pairs of inverse submoduls by

$$A = aK, \quad A' = a^{-1}K,$$

where K is a subfield and a is an element of F. If in particular a is an element of K, then A = A′ = K; if a^2 (but not a) is an element of K, then A = A′ is self-inverse, but is not a field. If a^2 is not an element of K, then A and A′ have no common element besides zero. The case of characteristic 2, needs a special discussion which is also given in the paper. Finally the theory is applied to the geometry of the Euclidean plane.

R. Vaidyanathaswamy, Madras.

Characterisation of the Quasi-Boolean Algebra as a Distributive Lattice.

The result is proved that the necessary and sufficient condition for a distributive lattice to be a Quasi-boolean Algebra is that every ideal containing a prime ideal be itself prime.

T. Venkatarayudu, Masulipatam.

On the linear Algebra of classes of elements in a finite Abelian group.

In my previous paper under the same title I defined C(d) as the class of elements of order d in a finite Abelian group G and showed that the classes C(d) combine among themselves by the group operation. Let the given Abelian group G be of order N and let d be a divisor of N. We denote by B(d) the totality of elements in G which can be expressed as the dth powers of elements in G but no higher power d' of any element in G, d and d' being divisors of N. The object of the present paper is to show that the classes B (d) combine among themselves by the group operation. This algebra defined by the classes B (d) will be found to be quite analogous to the algebra of the classes C (d). The two algebras will be identical if and only if the given group is cyclic in which case C (d) = B (N/d).

M. Zia ud Din, Aligarh.

Groups and Conjugate Matrix solutions of equations.

Recently Groups and Matrices have attained a prominent place in modern Physics and there is a wide field of research in them. In this Paper the Characters of the Symmetric group have been used to obtain the Conjugate Matrix solutions of certain equations. A general method is indicated.

K. S. K. Iyengar and K. V. Iyengar, Bangalore.

The solution of an Extremum problem in interpolation-theory.

In this paper we solve the following problem. Let $y = f(x)$, be such that $\overline{\phi} \geqslant D^n(y) \geqslant \underline{\phi}$, where $D^n(y)$ exists at all points except at a set of measure zero, $D^{n-1}(y)$ being continuous throughout a given interval (a, b), and $\overline{\phi}$ and $\underline{\phi}$ are summable. Suppose further that the values of y and its derivates up to $(n-1)$th orders are given at a and b. The first problem that arises is to find the conditions to be satisfied by the given extreme values in order that a y with the imposed conditions exist. The second problem is to find the upper and lower curves in case the values are consistent. These two problems are solved in this paper. The principal difficulty is to establish the existence of the extreme curves, which has been done by a somewhat unique method which is perhaps capable of being applied to a larger class of problems. We have generalised this problem, for the case when $D^n(y)$ is replaced by $D^n(y) - \sum_{1}^{n-1} D^r(y).K_r$, where $K_r \geqslant 0$, and have considered various particular cases. Incidentally we have proved the reality of roots of a class of algebraic equations which does not appear to be capable of being tackled directly.

K. S. Ramamurthi, Tambaram, Madras.

On Positive Valued Functions of a Real Variable.

A function $f(x)$ is said to be concave if $f\{(\sum x_i)/n\} \geqslant \{\sum f(x_i)\}/n$ and convex if the inequality is reversed.

$f^{\alpha}(x)$ denotes the function got by raising the original function $f(x)$ to the power α.

It can be shown that for positive valued concave functions the values α such that $f^{\alpha}(x)$ is concave is an interval of which the lower end point is zero. The higher end point α is called the *order of the* function.

Similarly for convex functions $F(x)$ there is a lower limit of numbers α for which $F^{\alpha}(x)$ is convex. This will be $\leqslant 1$. This will be called the order of the function.

The problem of extending the notion of order to general positive valued functions is also studied in the paper.

K. S. K. Iyengar, *Bangalore.*

1. *Generalisation of a problem of Erdos and Grunwald ;*
2. *A problem in function space ;*
3. *The existence of Frullani's Integral.*

C. N. Srinivasiengar, Bangalore.

Non-Differentiable Functions II.

Some further remarks are made on the properties of such functions, with particular reference to Peano's function.

S. Minakshisundaram, Madras.

On the Roots of a Continuous Non-Differentiable Function.

Let $f(x)$ be a continuous non-differentiable function defined in $0 \leqslant x \leqslant 1$, whose least and greatest values are L and U. If $S(\alpha)$ denotes the set of points x for which $f(x) = \alpha$, then for almost all values of α, $L \leqslant \alpha \leqslant U$, $S(\alpha)$ is non-enumerable and of measure zero. More precisely if

A = the set of points α for which $S(\alpha)$ is of positive measure,

B = the set of points α for which $S(\alpha)$ is of zero measure and non-enumerable,

C = the set of points α for which $S(\alpha)$ is at most enumerable, we have then the following

THEOREM :—

A is at most enumerable

$$|B| = U - L$$

$$|C| = 0$$

where $|X|$ = measure of the set X.

For Weierstrass' Non-Differentiable function $W(x) = \sum a^n \cos b^n x$, *C is not empty.* In fact S(L) and S(U) are enumerable. Also the proper maxima and minima of W (x) are the only values α for which S(α) can be enumerable, so that C is at most enumerable for W(x).

P. D. Shukla, Lucknow.

On the Derivates of a function of Denjoy

In the present paper the following function defined by Denjoy,

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{(n\,!)^{n-1}} \cos (n\,!)^n \pi x,\ 0 \leqslant x \leqslant 1,$$

has been considered. Denjoy used a geometrical method for proving the non-differentiability of the function and obtaining some of its properties. I have used an analytical method to determine the actual values of the derivates at points in (0, 1), and in particular have been able to find out the set of points where the function possesses one-sided infinite differential coefficients.

D. P. Banerji, Mymensingh.

On the properties of the functions which are self-reciprocal in Hankel's transform.

The following properties are a selection of results from the paper :—

(i) If $f(x)$ be R_u then $\frac{\sqrt{\pi}\ p^{\mu - \frac{3}{2}}}{2^\mu\ \Gamma(\mu + \frac{1}{2})}$ $\varphi\,(p)$ is $R_{\mu - 1}$ where $\varphi(x)$ is the operational image of $f(x)\ x^{\mu - \frac{1}{2}}$

(ii) If $f(y)$ be R_μ, the operational image of

$$y^{\mu - \frac{1}{2}}\ f(y) \text{ is } \frac{\Gamma(\mu + \frac{1}{2})\ 2^\mu}{\sqrt{\mu}}\ p^{-(\mu - \frac{3}{2})}\ g\,(p) \text{ where } g(p) \text{ is } R_{\mu - 1}$$

(iii) If $f(x)$ be R_μ then

$$\varphi\,(x) = a^\alpha\ g\left(xa^{2\alpha}\right) \pm \frac{1}{a^\alpha}\, g\left(\frac{x}{a^{2\alpha}}\right)$$

is $\pm$ R_ν when $a > 0,\ \alpha > 0$

V. Ganapathy Iyer, Annamalainagar.

On Maximal Integral Functions.

A "maximal integral function" $f(z)$ of order ρ and type d $(\rho > 0,\ d > 0)$ is defined as a function for which

$$\lim_{|z| \to \infty} \frac{\log |f(z)|}{|z|^\rho} = d$$

as $|z| \to \infty$ outside a system of circles the sum of whose radii is finite. If $n(r, a, \theta)$ be the number of zeros of $f(z) - a = 0$ in the sector common to $|z| \leqslant r$ and an angle of magnitude θ, I prove that $\underline{\lim}_{r \to \infty} \frac{n(r, a, \theta)}{r^\rho} \geqslant H(\theta)$ where $H(\theta)$ is a constant depending only on θ.

If, in addition, $f(z)$ has only simple zeros and $[z_n]$ be these zeros, the relation

$$\lim_{n \to \infty} \frac{\log |f'(z_n)|}{|z_n|^\rho} = d, \qquad \dots \quad (1)$$

holds, then I show that any function $g(z)$ of order ρ and type less than d satisfies the interpolation formula

$$\frac{g(z)}{f(z)} = \sum \frac{g(z_n)}{f'(z_n)} \frac{1}{z - z_n} \qquad \dots \quad (2)$$

Conversely, if a function $f(z)$ of order ρ and type d has only simple zeros and the relation (2) holds for all functions $g(z)$ specified above, then $f(z)$ is a maximal integral function for which (1) holds.

S. M. Shah, Aligarh.

A relation between the zeros and the maximum modulus of an Integral Function.

In this note I prove the Theorem :— If $F(z) = e^{g(z)} f(z)$ is an integral function of integral order $p \geqslant 1$ and $f(z)$ is a canonical product of genus p then

$$\underline{\lim}_{r \to \infty} \frac{\log M(r, F)}{n(r)\ \phi(r)} = 0$$

where $\qquad M(r, F) = \operatorname{Max}_{|z| = r} |F(z)|$

$n(r, F) = n(r, f) = n(r)$, the number of zeros of $F(z)$ in and on $|z| = r$ and $\phi(x)$ is any positive continuous, non-decreasing function of the real variable x such that

$$\int_\alpha^\infty \frac{dx}{x\ \phi(x)} \text{ is convergent.}$$

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R. S. Varma, Lucknow.

1. *Some infinite series involving Sonine's polynomial.*

In this paper I have discussed series of the type

$$\sum_{r=1}^{\infty} b_r \, T_r^n(x).$$

2. *Some infinite Integrals involving Bessel functions.*

In the present paper some infinite integrals involving Bessel functions are first investigated. Some interesting special cases are then discussed.

Hari Shanker, Delhi.

On an integral representation of Weber's parabolic cylinder function and its expansion in an infinite series.

The object of this paper is to obtain an integral representation of $D_n(z)$ in the form

$$e^{-\frac{1}{4}z^2} D_n(z) = -\frac{\Gamma(n-m+1)}{2ri} \int_{\infty}^{(0+)} e^{-\frac{1}{4}(z+t)^2} D_m(z+t)(-t)^{m-n-1} dt$$

holding for arbitrary values of m, and to expand $D_n(z)$ in an infinite series of the type

$$e^{-\frac{1}{4}z^2} D_n(z) = \Gamma(m+1)\,\Gamma(n+1) \sum_{r=0}^{\infty} \frac{(-1)^r D_{n-r}(z)\, D_{m-r}(z)}{\Gamma(n-r+1)\,\Gamma(m-r+1)\, r!}$$

S. Minakshisundaram, Madras.

On Partial Differential Equations of the Parabolic Type.

Using Fixpunktsatz of Schander, it can be shown, that we can find a function $u(x, t)$ expressible in the form

$$u(x, t) = \sum_{n=1}^{\infty} u_n(t) \sin nx$$

the series on the right converging uniformly and satisfying the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = f(x, t, u)$$

almost everywhere, in $0 \leqslant x \leqslant \pi$ and $0 \leqslant t \leqslant T$, and fulfilling the boundary conditions

$$u(0, t) = u(\pi, t) = 0, \; t \geqslant 0$$
$$u(x, 0) = u_0(x) \equiv 0.$$

provided $f(x, t, u)$ is a bounded measurable function continuous with respect to u and defined in the domain

$$0 \leqslant x \leqslant \pi, \; 0 \leqslant t \leqslant T, \; |u| < M.$$

If $f(x, t, u)$ is a monotonic increasing function of u, there can be only one solution.

K. V. Iyengar, Bangalore.

1. *Simpler proofs and some more theorems connected with Jordan-Brower theorem.*

The object of this note is to extend the method which the author used recently to give a simple proof of the Jordan-curve theorem to analogous theorems in three dimensions. Some additional results are obtained of which the following are cited as examples.

(i) The Smoothness (i. e. glatt-heif) property of one-dimensional cycles at any point of a Jordan-curve in three and higher dimensions.

(ii) If P and Q, two points on different boundary continua of a region say in 3-dimensions are joined by a Jordan arc running inside the region, then the one dimensional Betti-number is increased by unity.

2. *A Note on Poucelet's Problem.*

If P_1, P_2, $\cdots\cdots$ P_n, are points on a conic S' such that $P_r\ P_{r+1}$, $r=1, 2, \cdots n-1$ are all tangents to another conic S, then it is well known that $P_1\ P_n$ touches a conic of the type $S+\Delta_n\ S'=0$. The object of this note is to obtain the recurrence formula of Δ_n by simple geometric method without making any use whatever of elliptic functions.

3. *The projective interpretation of the ϕ-conic and corresponding covariants of two quadrics in higher dimensions.*

Various projective interpretations of the same have been given by several authors; but none of them, concern the case when the quadrics are given by their Polarities (according to the Von Staudt definition,) and in a geometry devoid of the continuity axiom (i. e. Pascalian geometry.) In this paper this problem is solved; e.g. the polarity corresponding to the ϕ-conic of two conics S_1 and S_2 is obtained as follows.

Let p be any plane. P_1 and P_2 their poles w. r. t. S_1 and S_2 respectively, P_{12}, and P_{21} *their* polars w. r. t. S_2 and S_1 respectively, and P_{121} and P_{212}, the poles of *these* w. r. t. S_1 and S_2 respectively. Then the polar of p, w. r. t. the ϕ-conic is the point of intersection of P_1P_{121} with P_2P_{212}.

M. N. Narasimha Iyengar, Mysore.

1. *On a problem connected with tetrahedra of given faces.*

2. *On conics having a common self-conjugate triangle.*

3. *On distances between special points associated with a triangle and a tetrahedron and their applications to the geometry of conics and conicoids.*

Karam Chand

Applications of Desargue's Theorem

V. Rangachariar, Patna.

On the nature of the conicoids through a quartic-curve and touching a given plane.

Three conicoids can, in general, be drawn to pass through a given quartic curve and to touch a given plane. The object of this paper is to discuss the nature of the three conicoids corresponding to a given plane. There are three paraboloids of the system two of which are elliptic and the third hyperbolic. Of the three conicoids touching a given plane one is always real and is a hyperboloid of one sheet. The nature of the other two conicoids depends on the position of the plane relative to the two elliptic paraboloids.

Ram Behari, Delhi.

Rectilinear Congruences whose foci are at infinity.

In this paper necessary and sufficient conditions that a rectilinear congruence may have its foci always at infinity are obtained. Some instances of congruences which have both foci at infinity have also been considered.

B. Ramamurti, Ajmere.

On a certain correspondence between lines in [4] and planes in [6].

In a paper, published recently by the author (A generalisation of the null pencil *Jour. London Math. Soc. Vol. 13, 1938, 162-167*) a correspondence is established between points in $[n+1]$ and certain special $[n-1]$s of $[2n]$. The object of this paper is to study in detail the specialities which follow in the case $n=3$.

A. Narasinga Rao, Annamalainagar.

On the Topology of line-elements in the inversive plane.

The connection between the group of a geometry (in the sense of Klein's *Erlanger Program*) and the topology of the operand-manifold arises from the need for a suitable closure by adjunction of ideal elements so as to secure that each transformation of the geometry shall effect a (1, 1) correspondence of the elements with themselves. It is shown that the oriented line elements of the inversive plane are topologically equivalent to the points of a projective 3-space while the non-oriented line elements are homæomorphic with a lenticular 3-space whose Poincare Group is cyclic and of order 4. Both these are of the type known as fibred spaces, the fibres corresponding to "stars".

P. Samuels Lall, Lahore.

On an extension of the method of least squares.

(i) The principle of the method of Least Squares is stated.

(ii) In certain cases it may happen that one of the quantities x_n to be determined, though unknown, is suspected of lying between certain limits. It then becomes necessary to determine the remaining unknown quantities in terms of x_n for a set of possible values lying within an interval and further considerations lead to the most plausible set of values of the unknowns.

(iii) A simple means for an extension of the Least square method is given for such cases. An example is given in which the above method is applied.

A. A. Krishnaswami Ayyangar, Mysore.

1. *A synoptic view of some small sample distributions.*

This paper expounds a derivation of the well-known distributions of the mean, the standard deviation, correlation-coefficient and a few other statistical derivatives of small samples by a uniform procedure based on linear transformations and Dirichlet integrals without appeal to n-dimensional geometry.

2. *Some problems on mathematical expectation.*

It was sometime ago noticed in our journal that the mathematical expectation of a drawing from a bag containing coins is

independent of the manner of drawing. We show here that this statement is not true in general and the apparent cases of success are due to fortuitous algebraic circumstances.

M. V. Jambunathan, Mysore.

The Moments of a Frequency Distribution employing 'Repeated Ogives'.

Let f_r denote the frequency of the occurrence of the value $r\,(r=1, 2, 3, \ldots . n)$ and let the "backward ogives" or frequencies summed cumulatively from the end f_n be denoted by ${}_1G_r$, i.e. let $\sum_r^n f_m = {}_1G_r$. Similarly, let

$$\sum_r^n {}_1G_m = {}_2G_r; \quad \sum_r^n {}_2G_m = {}_3G_r; \quad \sum_r^n {}_3G_m = {}_4G_r, \ldots \ldots \text{etc.}$$

In this paper it is shown that the several moments of the frequency distribution are given by the relations

$$M_0 = {}_1G_1; \quad M_1 = {}_2G_1;$$
$$M_2 = {}_3G_1 + {}_3G_2;$$
$$M_3 = {}_4G_1 + 4 \cdot {}_4G_2 + {}_4G_3;$$
$$M_4 = {}_5G_1 + 11 \cdot {}_5G_2 + 11 \cdot {}_5G_3 + {}_5G_4; \text{ etc.}$$

It will be noticed that the results obtained by the above method are symmetrical, a feature which the 'summation method' of computing the moments is lacking in.

M. R. Doresamiengar, Mysore.

Variation of Pareto's Law—a sequel.

"Pareto is a great name and analytical laws in Economics are few".........naturally, a Variant of Pareto's statement that is proposed, has to be expectant of criticism. The present paper is a continuation of the author's original paper, "Variation of Pareto's law" and seeks to explain the *a priori* reasoning behind the statement, $\frac{dN}{N} \Big/ \frac{dx}{x} = -kx$ and also sets out the mathematical consequences of other variants, like $\frac{dN}{N} \Big/ \frac{dx}{x} = -kx^2$; $\frac{dx}{x} \Big/ \frac{dN}{N} = -kN$.

M. V. Vaidyanatha Sastri, Hyderabad.

Heliacal rising of the moon

The moon's visibility for the first time in the evening after new moon determines the beginning of the Hijri month. This is of special importance in preparing the official Hijri calendar. The ancient Hindu methods based on the elongation of the moon, of determining the day on which the phenomenon occurs are discussed in this paper. It is shown that the altitude criterion is the safest one for deciding about the visibility of the moon and that her visibility can be taken as fairly certain when her altitude is greater than eleven degrees.

H. Subramani Iyer

Precession in Hindu Astronomy

B. S. Madhava Rao, Bangalore.

On the reduction of dynamical equations to the Lagrangian form

It is shown that in the case where the kinetic energy is quadratic in the velocities the condition for the reduction to Lagrangian form is deduced very simply by using tensor methods. Other cases are also considered, and the application of the method of the last multiplier to the problem is investigated.

2. *On an invariant relation of dynamical systems*:

A. Wintner (*Q. J. Math.* Vol. 7, (1936), p. 214) has derived a new elementary invariant relation valid along those solutions of certain dynamical systems which run in the sub-space $h=0$ of the phase-space. The full significance of this in the general case from a geometrical point of view is discussed in this paper.

K. Nagabhushanam, Waltair.

On a Property of the Lagrangian.

In an affine manifold of states and time, following the invariant definition of the Lagrangian given by me, I here show that for the singular curves of the curl of (X_i),

$$\iint \cdots \int \mathrm{L}\ \alpha^{2n+1}\ d\omega$$

is an integral invariant for all co-ordinates with $t = x^{2n+1}$, and that when

$$x^r = q^r, \quad x^{r+n} = p_r, \quad x^{2n+1} = t,$$

this takes the form

$$\iint \cdots \int \left(p_r \frac{\partial \mathrm{H}}{\partial p_r} - \mathrm{H} \right) dq^1\ dq^2 \cdots dq^n\ dp_1\ dp_2\ dp_n\ dt.$$

B. R. Seth, Delhi.

Strain in a spherical shell turned inside out.

The problem of a spherical India-rubber shell turned inside out is one which cannot be solved by the ordinary Mathematical theory of Elasticity which deals with only small strains. In the present paper the theory of Finite strain which has been developed by me in some recent papers* is applied to this problem, and the following results have been obtained:

(i) The increase in the external radius of the shell is about 2 per cent.

(ii) The thickness decreases by about 0·07 per cent.

(iii) The radial stress $\widehat{rr}$ does not vary very much with r.

(iv) The absolute value of the cross-radial stress $\widehat{\theta\theta}$ is greatest at the inner boundary.

B. Ramamurti, Ajmer.

A theorem on Eddington's E numbers, and its relation to Dirac's theory of the electron.

Four numbers E_1 E_2 E_3 E_4 so that $E_r^2 = -1$ and $E_r E_s = -E_s E_r$ $(r \neq s)$ generate 16 linearly independent numbers including -1. Any number A formed from a linear combination of them is said to be singular if another number B could be found so that $AB = 0$. The totality of E numbers, which form a linear ∞^{16} system, and hence may be said to be of rank 16 is reduced to a lower rank by multiplication by a singular member. The object of this note is to prove that the minimum rank to which the system of E numbers could be reduced is 4.

The E numbers are equivalent to the 16 numbers generated by the four symbols introduced by Dirac, in his relative equation of the electron. Sauter† has solved Dirac's equation without any matrix representation of Dirac's symbols, by taking any function of the 16 symbols and reducing it to rank 4 by multiplying by a properly

* *Phil. Trans. Roy. Soc.* A, 234 (1935), 231-64; *Proc. Roy. Soc.* A, 156 (1936) 171-92 *Phil. Mag.*, Ser. 7, 27 (1939), 286-93.

† Sauter: *Zeitsohrift fur Physik*, vol. 63, pp. 803 (1930)

chosen symbol. Our result shows that it cannot be reduced further. Probably this explains why Dirac's state function necessarily involves four components.

Gunjikar and **Ozu**, Bombay

Joules Law of minimum dissipation.

The paper shows the invalidity of the conclusions usually drawn from the minimum property of the expression $\sum C^2R$ for a network of steady currents.

A. H. Querishi, Peshawar.

The case for and against mathematics.

A. N. Singh, Lucknow.

A Sanskrit Translation of Euclid.

A Sanskrit Translation of Euclid was made in 1718 A.D. by Samrat Jagannatha, the court-astronomer of Maharaja Jai Singh of Jaipur. This translation has been published by K. P. Trivedi. The translation is based on an Arabic version. Trivedi thought that the Sanskrit translation was made from the Arabic version of Nasir-uddin, but a closer examination of the work reveals that the Sanskrit translation is based on the Arabic version of Ishaq. The present paper gives a short account of the life of the translator, Samrat Jagannath, and of some interesting points connected with his translation.

V. P. Venkatachary, Hyderabad.

India's Contribution to continued fractions.

An attempt is made in this paper to show how the ancient Hindu Mathematicians originated the theory of Continued Fractions. Arya Bhatta (born 476 A.D.) was the inventor of this theory. His method of successive reductions corresponds to the method of Continued Fractions. Later mathematicians followed him with improvements and extensions. Bhaskara's method of solving linear indeterminate equation is more akin to the modern method. Brahmagupta originated the theory of recurring continued fractions by the enunciation of his remarkable principle known as 'Principle of

Composition of roots.' He could solve $x^2 - Ny^2 = 1$ (N being a non-square integer) when he knew the solutions of $x^2 - Ny^2 = k (k = \pm 2, \pm 4, -1)$.

Bhaskara tried to solve $x^2 - Ny^2 = 1$ when the solutions of $x^2 - Ny^2 = k$ (any integer) were known to him. In this connection he stated his famous method 'Chakravala' which embodies the whole theory of recurring continued fraction. The important properties of recurring C.F.'s are deduced from 'Chakravala' or the 'cyclic method'. It is shown in the end that Bhaskara's method of solution is very simple and claims superiority to any other method, ancient or modern, in the matter of practical computation. A few examples are worked out to elucidate the various points.

GLEANINGS

THE CIRCLE

By Christopher Morley

Few things are perfect : we bear Eden's scar ;
Yet faulty man was godlike in design
That day when first, with stick and length of twine,
He drew me on the sand. Then what could mar
His joy in that obedient mystic line ?
Approximating with a zeal divine
He called π 3-point-14159
And knew my lovely circuit $2\pi r$!

A circle is a happy thing to be—
Think how the joyful perpendicular
Erected at the kiss of tangency
Must meet my central point, my avatar.
And lovely as I am, yet only 3
Points are needed to determine me.

[From "*Scripta Mathematica*" October 1938.]

Symposium on Generalized Geometry, including Relativity and Field Theories

Chairman: PROF. D. D. KOSAMBI, *Fergusson College, Poona*

Initiating the Symposium, the Chairman said :—

GENTLEMEN !

This symposium is intended to be a set of twenty-minute lectures by six people on their own or connected work in branches of modern geometry, mainly the kind of geometry associated with tensor analysis. We are not in the fortunate position of Dr. Pillai, whose symposium deals with a single clearly defined and well delimited topic—Waring's problem ; a problem which is almost finished because of his own efforts though regarded as almost insoluble ten years ago ; and yet a problem which has enough interest left to make it ideally suitable for the purpose of a symposium. Today's lectures will be on tensor analysis, differential invariants, group theory, and topology.

You will then wonder whether it was worth while to tie together so many scattered branches of a vast subject. As a matter of fact, there is a profound fundamental unity in all that will be said today, and the lifework of one very distinguished mathematician has gone to show this unity.

I refer to Prof. Elie Joseph Cartan of Paris, who completed his seventieth year in April, and to whom I propose—with your permission—to dedicate this symposium in appreciation of his magnificent contributions to science. I trust that you will agree with me in the hope that the tribulations of the war in which his country is engaged will not prevent his putting the finishing touches to a pioneer work which constitutes perhaps the most important individual performance in differential geometry in our time.

Let me add that work in geometry in India is not confined to the topics we have chosen. You must all have heard of the work in algebraic geometry performed by Dr. Vaidyanathaswamy and his pupils ; also, the work on topological differential geometry, after Blaschke, so brilliantly carried out by Prof. Shyamadas Mukhopadhyaya and his co-workers. Lastly, the relativitists and field

theorists deserve, and originally had, a symposium to themselves. This was cancelled because of the crowded program, but you will hear two of the best today, and Dr. N. R. Sen has inaugurated the conference, so that we have not lost much after all.

The Chairman then moved and conference passed a unanimous resolution of sympathy for Prof. Tullio Levi-Civitá of Rome, who, at the end of a long and distinguished mathematical career, is now suffering from the newly developed anti-semitic policy of the Italian Government. The Symposium itself was dedicated to Prof. Elie Joseph Cartan of Paris in honour of his seventieth birthday.

Prof. KOSAMBI then proceeded to show how simple the elementary notions of tensor analysis can be when taken without any of the formal apparatus of algebraic invariant theory. Starting with the transformation laws for covariant [force-like] and contravariant [velocity-like] vectors, we can develop a tensorial differentiation and calculate all the differential invariants associated with a system of differential equations of the second order. As a special case, one can consider the systems involved in the theory of relativity. These can be generalised in one direction by considering all the systems that admit the Lorentz group of transformations. It was shown that systems of second order differential Equations of this latter type not only include all equations proposed by Milne, but also make clear what the utmost possibility is as regards an "intrinsically expanding" universe. [the technical portions of the lecture will be published in a forthcoming issue of the *Journal of the Lond. Math. Soc.*]

DR. B. S. MADHAVA RAO (Bangalore) read an elaborate paper † which dealt with the intimate connection existing between *Generalised geometry and physical theories.* The old idea that physical space is identical with three-dimensional Euclidean space has given place, with the advent of Relativity, to the notion that geometry, in so far as it introduces the determination of measure in the space-time world, has essentially an empirical character. Thus the notions of Riemannian Geometry have become of fundamental importance in physical attempts at geometrisation of natural phenomena. With the appearance of Quantum Mechanics the geometry of abstract spaces, specially the Hilbert space, has come to have various applications in physics. In some recent theories devised to get over the fundamental

† This will appear in the next issue of the STUDENT.

difficulties of quantum mechanics, one can see a trend towards ideas of non-Riemannian geometry and topology.

Even the old classical mechanics has been influenced by the newer geometrical notions. Thus the notions of configuration space and phase space have led to the theorems of classical mechanics as geometrical theorems in Riemannian space. The properties of dynamical systems "in Grossen" have led to topological characterisations of dynamical motions.

The paper dealt with several such theories drawn from Relativity, Cosmology, electro-magnetic field theory, Quantum Mechanics and Nuclear Physics.

FATHER C. RACINE (Madras) then spoke on *Group Theory and Generalized Geometries*. A summary of his address is given below :—

Prof. E. Cartan has shown how it is possible to develop *any* differential geometry upon the basis of the theory of finite transformation groups. This basis is by far the most natural and the broadest possible. It has proved also to be the best guide in investigations about the possible generalizations of Geometry and the *complete* set of differential invariants, if not their analytical expressions, in each case. This lecture aims at outlining the ideas and methods of Prof. E. Cartan.

In the first part, the notion of fundamental space and of the moving frame of reference—" Repère Mobile "—is dealt with, along with a method to find out the differential invariants of any variety embedded in such a space.

In the second part the notion of "differential connectivity" is defined, and a method for generalizing any fundamental space developed. Exactly as a sphere can be considered as the limit of a polyhedron, so any generalized space can be considered as the limit, under certain conditions, of a certain assemblage of small portions of a fundamental space. A few examples such as spaces with euclidean, projective, conformal, ... connectivity are discussed. It is shown how all their differential invariants may be deduced from the equations defining the differential connectivity.

A complete bibliography has been published in the Jubilee Volume "Selecta" (Gauthier-Villars 1938). A partial one, but quite sufficient for a first contact with these fields of Research, is to be found in Cartan's Volume " La théorie des groupes finis et continus

et la Géométrie différentielle traitées par la méthode du Repére Mobile" (Paris, Gauthier-Villars, 1937). Reference may be made also to the fascicle 42 of "Mémorial des Sciences Mathématiques". It deals with the theory of finite and continuous groups from a new point of view but may be taken as a very good introduction to the subject.

Prof. N. G. SHABDE (Nagpur), spoke on *Projective Relativity*. The problem of unified field theory is to find a geometry, representing both gravitational and electromagnetic phenomena. A generalized theory making use of five homogeneous co-ordinates is given by Schouten and van Dantzig. Introducing analogues of tensors, called projectors, the projective curvature tensor $N_{\dot{p}\dot{q}\dot{r}\dot{s}}$ is constructed. From this the scalar N is obtained by methods similar to those in ordinary tensor-calculus. The variational principle $\delta I = 0$ where

$$I = \iiiint\!\!\int N\sqrt{G}.\ dx^0\ dx^1\ dx^2\ dx^3\ dx^4$$

gives the gravitational and electromagnetic field equations. As has been shown by the lectuier (*Phil. Mag.* Vol. XXII), an application of a variational theorem by Emmy Noether to I gives the indentities between the field equations in the field theory of Schouten and van Dantzig.

Prof. V. SEETHARAMAN (Annamalainagar) who spoke next outlined briefly two *Methods of generating differential invariants,* the one due to Prof. D. D. Kosambi and the other to Prof. Elie Cartan of Paris. For illustration he considered a path-Space of order two, viz. one in which the paths are defined by differential equations of order three,

$$x^{(3)i} + \alpha^i\ (x,\ x',\ x'',\ t) = 0.$$

The first method referred to, consists in first deriving a basis for all invariantive operators of the space, and then by alternating these taken two at a time, all the invariants are obtained. That the invariants so obtained furnish the basis for all the invariants of the space can be seen even by a direct appeal to the equations of the paths and their equations of variation. The invariants should be the derivators of α^i of various orders. The first derivatives are in essence the primary tensors appearing in the equations of variation and further derivatives lead to $\gamma^i_{j(2)k}$ and $\underset{\mathrm{I}}{\nabla_k}\underset{\mathrm{I}}{\nabla_j}\gamma^i_r$ as the only invariants of the second and third rank respectively.

Cartan's method consists in calculating the Poisson parentheses of certain Pfaffians, that are defined in terms of dx^i, $d\dot{x}^i$ and $d\ddot{x}^i$ and another which gives us the connection.

He concluded by pointing out that both these methods gave us the same set of basic invariants, a result naturally to be expected since both the methods reduced to alternations of fundamental operations.

Prof. K. VENKATACHALIENGAR (Bangalore) concluded the Symposium with a talk on *Point Set Topology* or *Continuity Geometry*, a subject whose foundations had been laid by Cantor but which had developed remarkably in recent times. One problem is to define space in a topologically invariant manner with a minimum set of axioms. From generalised topological space we derive the Hausdorff-Kuratowski, Frechet, Hausdorff, regular, normal and metrical spaces. The last among these has been topologically characterised by the property that "A regular space (*i.e.* one in which every neighbourhood of any point P contains a closure of another neighbourhood of P) with an enumerable basis is metrisable".

As another example may be given Lebesgue's definition of the "dimension" of a set: a set A is n-dimensional if every covering of A by means of closed sets (of arbitrary fineness) contains at least one point common to $(n+1)$ of them.

The characterisation of a Jordan curve in an R^2 by means of its outer properties is as follows: It must divide R^2 into exactly two regions and all points of the set must be reachable boundary points for both the regions. Brouer has shown by means of examples that if the points of the set are not reachable, the points need not form a Jordan curve.

Symposium on Waring's Problem

Chairman: PROF. S. S. PILLAI, (*Annamalainagar.*)

Giving a historical survey of Waring's Problem, Dr. Pillai said:—

The Theory of Numbers is the only branch of mathematics in which many of the problems solved and unsolved may be explained to laymen, and even broadcasted in newspapers. It is no surprise that Vinogradow's sensational result, that every large odd number is the sum of three primes, got a place in the *Sunday Times*. In no other branch of Mathematics, can one find problems which are more than

two-thousand years old and yet defy all attempts to solve them. Many of the majestic peaks of Number Theory are such that any one can have a glimpse at them, but even the greatest masters of the science have no hope of reaching them at present.

Two of such outstanding peaks are 'Waring's problem' and 'Goldbach's theorem', named after the men who had the first glimpses of them. Many are the attempts made to climb up to these. It is really astonishing that a single man's partial success has eclipsed the results obtained by all others in both these problems. Today, Vinogradow has the glory of reaching heights which are marvellous compared to those conquered by all others. Why, in Goldbach's theorem he has reached a peak which is as important as the main one! Though he has not reached the final goal in Waring's problem, the height that he has reached is amazing, and this has enabled other investigators to reach a subsidiary peak.

In the year 1640, Fermat stated, among other results, that every number is the sum of four squares. Though his statement implied that he had a proof by his famous method of infinite descent, it was Lagrange who published a proof for it in 1770. Suppose we want to express n as the sum of squares, say, in the form

$$n = x_1^2 + \cdot \cdot \cdot + x_s^2.$$

To start with, we have no reason to expect that the minimum value of s will not increase with n. Fermat's theorem means that it is possible to find s independent of n, and that its value is 4. This is certainly one of the most beautiful results in Arithmetic.

Waring's Problem is the generalisation of this result for k-th powers. In the year 1770, Waring, Lucasian Professor of Mathematics in the Cambridge University, stated, in his 'Meditations Algebraic,' that every number is the sum of 4 squares, 9 cubes, 19 biquadrats and so on. Let $g\ (k)$ be the least value of s such that every number is the sum of s k-th powers; *i.e.* the equation

$$n = x_1^k + x_2^k + \cdot \cdot \cdot + x_s^k$$

is solvable for every n. Here also, in the first instance, we do not see whether the least value of s will increase with n or not. Waring's assertion means not only that $g\ (k)$ is independent of n, but also that $g\ (2) = 4$, $g\ (3) = 9$, $g(4) = 19$, and so on. But Waring's result was only a conjecture, and he had no proof for it.

In 1772, Euler, the son of the famous Leonhard Euler, showed that $l \cdot 2^k - 1$ requires $2^k + l - 2$, kth powers, where $l = [(3/2)^k]$, and thereby

proved that $g(k) > 2^k + l - 2$. The next advance was made in 1858 when Liouville proved that $g(4) \leqslant 53$. In the years up to 1909 upper bounds for $g(k)$ were evaluated and improved for all k up to 10 excepting 9.

The year 1909 is an important one in the history of the problem. In that year, Wieferich proved that $g(3) = 9$. Landau proved that every large number is the sum of eight cubes, and above all Hilbert proved the existence of $g(k)$. Speaking about the problem and its solution by Hilbert, Professor Hardy says "Within the limits which it has set for itself, it is absolutely and triumphantly successful, and it stands with the work of Hadamard and de la Vallee-Poussin, in the theory of primes, as one of the landmarks in the modern history of the theory of numbers."

Landau's result is important in that it was responsible for the introduction of another and more fundamental function $G(k)$ by Hardy and Littlewood. Landau's 'singularly beautiful theorem' means that the number of numbers which require nine cubes is finite. As a matter of fact, recently Dickson has proved that 23 and 239 are the only numbers which require as many as 9 cubes. $G(k)$ is defined as the least value of s such that every sufficiently large number (every number from a certain integer onwards) is the sum of s, k-th powers. Since numbers of the form $8m + 7$ cannot be expressed as the sum of three squares. Fermat's result means that $g(2) = G(2) = 4$. Landau's result takes the form $G(3) \leqslant 8$.

Hilbert's proof was a pure existence proof. The next advance was due to Hardy and Littlewood. In the years from 1919 to 1927, they attacked the problem in a series of memoirs. They discovered a method of applying Cauchy's theorem to the problem. It may be remarked that the 'Farey-dissection' of the circle, which plays a fundamental role in all subsequent investigations, first appears in Hardy-Ramanujan's memoir on Partitions. By their important method, besides finding upper bounds for $G(k)$, Hardy and Littlewood were able to find asymptotic formula for the number of representations of a number as the sum of s k-th powers, when s is large. In these memoirs, they introduced many fundamental ideas and devices, without which subsequent advances would have been impossible. The final result about $G(k)$ which they proved is that

$$\overline{\lim}\ G(k) / (k \cdot 2^k) \leqslant \tfrac{1}{4}.$$

Though the success of their method was phenomenal in dealing with $G(k)$, they did not apply it to obtain an upper bound for $g(k)$.

But Vinogradow found out a new method to prove Hilbert's theorem, and the consequences of that method are given in Landau's *Vorlesungen über Zahlentheorie.* The result proved there, is that

$$\overline{\lim}\ g(k) / (k \cdot 2^k) \leqslant \tfrac{1}{2}.$$

In 1933, R. D. James applied the Hardy-Littlewood's method and reduced the constant in the above limit to 1/4 and improved upon the upper bounds for $g\ (k)$, given by earlier investigators for small values of k.

In the year 1935, Vinogradow was able to prove the wonderful result, that

$$\lim\ G(k) / (k\ \log\ k) \leqslant 6.$$

Though this is a tremendous improvement upon Hardy and Littlewood's result, it is far from the final solution of $G(k)$. Yet it prepared the ground for the solution of $g(k)$, which is final in a sense.

By evaluating the constant in Vinogradow's paper, I was able to prove, in September 1935, that*

$$g(k) = 2^k + (3/2)^k + O(4/3)^k.$$

The result was communicated to the conference of the Indian Mathematical Society held at Delhi in December 1935. But in the third week of December 1935, I was able to prove that

$$\text{if } \{(3/2)^k\} \leqslant 1 - (l+3)/2^k, \text{ then } g(k) = 2^k + l - 2, \text{ when } k \geqslant k_0.$$

The result was published in the March issue of *Journal of the Annamalai University*, and advanced reprints were sent to some mathematicians including L. E Dickson about the 10th February 1936 (perhaps, before the 10th).

By refining the above method, I brought down k_0 to 7 in the last week of January 1936, and published the proof when $k_0 \geqslant 8$ in the Journal of the Indian Mathematical Society. The fact that this problem should depend upon the fractional part of $(3/2)^k$ was a great surprise to me.

The proof of the above result when $k_0 \geqslant 7$ together with an exact formula for $g(k)$ when $\{(3/2)^k\} \geqslant 1 - (l-1)/2^k$ was published by L. E. Dickson in the July (1936) issue of the *Journal of the American Mathematical Society.* The latter result was discovered independently by me.

* It may be mentioned that the result was proved without applying Dickson's method of ascent.

These results form an 'almost complete solution' of the problem of $g(k)$ in a sense. Though there is no doubt that $\{(\frac{3}{2})^k\} \leqslant 1-(l+3)2^k$ for all $k \geqslant 4$, unless it is settled one way or the other, we cannot say that the problem of $g(k)$ is completely solved.

SAMBASIVA RAO (Waltair) said that Vinogradow's success in Waring's Problem was partly due to successfully using the following function: Let $H_{s,k}(N)$ denote the number of integers $\leqslant N$ that are expressible as the sum of s k-th powers. By considering integers representable as the sum of s k-th powers in a particular fashion, he proved that $H_{s,k}(N) > C.\ N^{1-\frac{1}{6k^2}}$ for $s \geqslant c_1 k \log k$. Any improvement of this inequality leads to an improvement in Vinogradow's result. Attempts are already being made by Davenport and others in this direction. Davenport's results lead to startling improvements in Waring's Problem in the case $k=3$ and 4. Continuing Davenport's work, the speaker was able to improve previous results in the cases $k=5$, 6, 7.

The solution of Goldbach's problem for odd integers by Vinogradow opened up a new field. It was immediately proved that every sufficiently large integer is expressible as the sum of $s\ \{=6\ (k \log k)\}$ k-th powers of primes. Following Davenport's lead improvements could be effected in this field also for small values of k.

Another direction in which Waring's Problem was generalised was the representation of integers as sums of polynomial summands instead of as sums of k-th powers, where the variable takes integer values, which may be required to be all primes. Work has been done in this direction by Hua, Pillai, the speaker and others.

Dr. T. VIJAYARAGHAVAN (Dacca) spoke mainly about the work of Dr. Chowla. The following were the main results to which he referred: (1) The existence of numbers which can be expressed as the sum k, k-th powers in any number of ways. (2) The expression of a number as the sum of almost equal k-th powers. (3) If K (x) is the number of numbers $k \leqslant x$ for which $g(k)=2^k+l-2$, then $\overline{\lim}\ K(x)=1$ and (4) Mr. I. Chowla's result on the addition of residue classes.

Discussion on the Teaching of Mathematics

Chairman: QAZI MOHAMMAD HUSAIN Esq. HYDERABAD

Prof. J. MACLEAN (Bombay) in initiating the discussion stressed the importance of certain topics which should receive special attention in a first year college course. These were, in his opinion, the function concept and the

functional scale, generalised coordinates, a fuller treatment of curves, the calculus, an idea of things in three dimensions, nomograms and statistics. As things stood at present, the same subjects as in the school course e.g. Algebra, Geometry were continued in much the same way, and little was done to give a new and wider outlook such as was required in the modern world. The criterion which ought to decide the inclusion of a particular topic should be whether it was far reaching, serviceable and unifying. The students should become familiar with the use of the slide rule and learn both the power of the graphical method and its limitations.

Prof. KISEN CHAND (Hyderabad) who followed was against the existence of alternative courses in Intermediate Mathematics, and said that there should be greater uniformity among the different Universities so as to make migration easier. He was for speeding up the existing courses so that by the end of the matriculation stage the whole of Euclid (11 books), progressions and Quadratics would have been finished. The two years of the Inter course provided about 300 teaching hours in which most of algebra, Trigonometry, Calculus, coordinate geometry, mechanics and hydrostatics could be covered. He then outlined the corresponding continuation courses in the degree and post graduate classes.

Prof. MOHD. KHAJA MOHIUDDIN (Hyderabad) deplored the wastage of time and effort in the high school stage and said that topics were sometimes taught in the wrong order, the difficult ones first and the simpler later. He cited by way of example the following problem "If $x + \frac{1}{x} = 1$ what is $x^2 + \frac{1}{x^2}$?" which, to the boy who knows nothing of imaginary numbers, started with an impossible hypothesis. He was for a rearrangement of the syllabus on psychological lines.

Prof. B. RAMAMURTI (Ajmere) pointed out that in nearly half the Universities in India calculus was taught in the Intermediate stage and that there were wide differences in standard even among these. In the matter of actual teaching he was for taking full advantage of the students' intuitive ideas and would begin a study of functional relationships and rates of growth on a foundation of graphical work. No loose statements should be made which the student would have later to unlearn. In other words, the method should be rigorous without an exhibition of rigour—rigorous in the long run. In the M. Sc. courses he was for stressing the Group concept and for the inclusion of topics like spherical harmonies.

Prof. C. R. CHATURVEDI (Agra) spoke on the disabilities of Mathematics students at the competitive examinations, a striking evidence of which was that such candidates very often offered a different subject at the competitive examinations. He made out a strong case for candidates offering mathematics at the Indian Accounts and Audit Service Examination being permitted to offer also Elementary Mathematics pointing out that a similar situations holds regarding other subjects and that the marks obtained in Elementary Mathematics by those offering subjects other than Mathematics were already so high that mathematics students could not do much better. He drew attention to the hiatus between the courses of study for the University examinations and competitive examinations. He was also for mathematics being placed on a par with History and other subjects in the matter of marks and for the general knowledge test being placed on the same conditions as the London I. C. S.

After a further discussion it was resolved that "The Managing Committee of the Indian Mathematical Society be requested to urge on the Federal Public Service Commission the need for investigating the various handicaps under which candidates offering mathematics in the Higher Competitive Examinations labour under existing regulations."

Dr. R. VAIDYANATHASWAMI (Madras) said that what was taught in Universities was "word knowledge." It would become Real knowledge only when it was seen in its proper setting with its experience in the world of life. Prof. Maclean's idea was that there should be courses which should catch such deep ideas as the mathematician has to offer for the enrichment of life without going too deep into life—courses fitting a man for a general cultural life. Naturally such a course would deal with topics connected with generalised coordinates, real number, function concept, rates of growth and statistics.

Prof. GUNJIKAR (Bombay) did not like uniformity and felt that each University should make its own experiments so as to allow for the diversity of tastes and interests. He discussed also the purposes served by examinations and how questions should be set so as to serve these purposes.

The Chairman Prof. QAZI MOHAMMAD HUSAIN (Hyderabad) in winding up the debate pointed out the importance of admitting students who had an aptitude for the subject. He described the outlines of a syllabus for the various grades of study in a University.

Minutes of the Business Meeting

held at 12-30 P.M. on 23—12—39.

Chairman:—DR. N. R. SEN, *Ghosh Professor of Applied Mathematics, Calcutta University.*

1. The financial position of the Society was considered and attention was drawn to the need for enlisting more members and subscribers.

Regarding the rates of subscription for members the general body suggested to the Managing Committee an examination of the life composition fees and the consideration, in particular, of the proposal for a uniform rate of Rs. 10/- per annum for all members and a life composition fee of Rs. 150/-

2. A committee consisting of Dr. N. R. Sen, Dr. Vaidyanathaswami, Dr. A. N. Singh with such others as they may co-opt was appointed to consider means for securing a closer co-operation between the Benares, Calcutta and the Indian Mathematical Societies.

3. The question of holding the conferences of the Indian Mathematical Society along with the sessions of the Indian Science congress was considered, but the suggestion received little support from the members of the Society.

Public Lecture on "Meteoric Astronomy"

BY

MOHD. ABDUL RAHMAN KHAN ESQ., F.R.A.S., *Hyderabad*

Meteoric Astronomy, as the name implies, is that branch of Astronomy which deals with the study of meteors. It endeavours to account for their light effects, their nature and origin in space. It involves a study of meteorites as well, which are closely allied to them, being fragments of matter dropped from larger and much more imposing meteors, that have penetrated through the entire depth of our atmosphere and survived its devastating effects. It is thus evident that meteoric astronomy has to seek aid, more or less, from a number of sciences other than astronomy, chiefly, physics, chemistry and geology.

Meteoric Showers

Though meteors have been observed from time immemorial and commented upon by scientists and soothsayers alike, their systematic study commenced only recently and has been occasioned by the interest created in them after the spectacular showers of 1799 and 1833 A.D. In both these showers the radiant, or starting point common to all the meteors of the class, was observed to be close to γ Leonis, hence their designation, Leonids. The former of the above two showers, which occurred on November 12 was carefully recorded by Humboldt while travelling in Cumana in South America; and the latter, a display of surpassing brilliancy, on November 13, by a number of observers in Europe and America. Denison Olmsted and Palmer of New Haven, Con. made a special study of it. According to Olmsted's estimate, no fewer than 240,000 meteors appeared in 9 hours; and Palmer connected the shower with that of 1799, as a recurring phenomenon with a period of roughly 33 years between two consecutive maximum displays.

In this way the Leonid shower was expected to repeat in 1866 and was hailed with great enthusiasm all over the world, when the Leonids began to appear in greater and greater numbers every year from 1864 to 1866. W. F. Denning described the display of November 14, 1866 as the finest observed in England, with 100 meteors per minute. The shower was repeated on the same date in 1867 and subsequent years, but gradually fell off by 1869. After this, H. A. Newton and other investigators ransacked old historical records and prepared a list of conspicuous Leonid showers of past epochs. Newton's list begins with the shower of October 13, 902, long remembered in the Arabic world as "the year of stars", from the fact that the Aghlabid Sultan, Ibrahim ben Ahmed died on the night of the shower, when the Saracen army had just captured Taoramina in Sicily.

[Calculation shows that when the Leonids passed in the neighbourhood of Uranus in 126 A. D., in pursuit of their former parabolic orbit, their path was deflected by the perturbation of that planet into an elliptic one, and their direction of motion round the Sun was completely reversed; so they nowadays approach the Sun to meet the Earth, with an average velocity of 80 kms. per sec. and a period of 33·25 years. The shape and dimensions of this orbit have remained practically intact since that event, but it has undergone a slight

shifting of position in its own plane, and the date of maximum display of the meteors is delayed by one day every 70 years.]

It is a pity that the next expected Leonid shower at the beginning of the present century turned out to be disappointing—as also that of the last (1933) epoch. But interest in Meteoric astronomy is now too well established to be affected by the waywardness of sensational showers. Thanks to the labours of Charles P. Olivier (Director, Flower Observatory, University of Pennsylvania) and the team work of the A. M. S. (an institution of international character) visual observations are conducted throughout the year over a vast range of the globe, and are published in the Meteors section of *Popular Astronomy*, Northfield, Minn. The enthusiasm and resources of the Harvard College Observatory under its present Director, Prof. Harlow Shapley and his staff, have enriched meteoric astronomy with methods of the finest accuracy of research, through photography and spectroscopy, and have led to valuable discoveries about the role of meteors in interstellar space, and the density and temperature of the upper regions of our atmosphere inaccessible to balloons manned or unmanned. As a result of these activities, it would be no exaggeration to say that almost every educated man in the United States of America has now become 'meteor-minded.'

The International Astronomical Union has also done much to encourage research in Meteors. No account of the subject would be considered complete without paying tribute to the work of the great pioneer, W. F. Denning. The work started by him is carried on by the B. A. A. in England. Very valuable information is published from time to time in the Research Notes and other sections of *Nature*, on meteoric astronomy. Some of the back numbers of this great weekly are a veritable mine of information about important meteoric showers and fire-balls. Excellent statistical and observational work is being done by Hoffmeister in Germany : by Millman in Canada : by S. Orlov and I. Astapowitsch at Moscow, by Hideo Inouye, Kozira Komaki and members of the Astronomical Society in Japan.

Apart from the Leonids, many other periodic showers have been identified and studied, some of which have been traced, through old Chinese Annals, to as early a time as 687 B.C. Many of these showers are derived from the débris of matter revolving in the orbits of well-known comets ; for example, the Leonids are associated with Tempel's Comet of 1866 I ; the Andromedes with Biela's (last conspicuous shower occurring in 1885, Nov. 27 in the Third Burmese War) ; the Perseids with Tuttle's 1862 III ; the Lyrids with Thatcher's 1861 I ; and the Giacobinids with that discovered by Giacobini-Zinner. It may be mentioned in passing that a grand shower of the Giacobinids was expected on October 9, last year, but it failed completely. Very likely it may occur on October 10, this year (1940.)

[A comprehensive list of notable meteoric showers by the present writer, published in the *Journal of the Osmania University* Vol. III 1935, may be consulted for further reference.]

The Society for Research on Meteorites established in 1933 with Headquarters at Denver, Colorado, U.S A., has given an immense impetus to the study of Meteorites, and has brought to light a number of Meteorite falls and finds that would otherwise have been lost to the scientific world.

Meteoric Matter in Space

The density of interstellar space is reckoned by Eddington to be about 10^{-30} grams per c.c. Apart from the matter constituting the stars the amount of loose material or 'dust' in the universe is by no means inconsiderable. C. C. Wylie estimates a fall per hour, of about a million meteors, visible to the naked eye, all over the world. The number of telescopic meteors swells up to several millions of millions per diem.

[Spectroscopic evidence deduced from the presence of stationary H and K lines of ionised calcium, and of D_1 and D_2 and other Sodium lines, not to speak of two titanium lines and a neutral potassium line, with several others still unidentified, in the spectra of stars moving with large velocities in the line of sight, supports the view that interstellar space is by no means so devoid of material particles as the older astronomers were led to believe.]

Harvard College Circular No. 317 estimates a fall of at least a million million meteors *per second* in the atmospheres of some of the brightest stars, as revealed by the presence of Cyanogen bands, Swan spectrum and Raffety bands in their spectra; and 'guarantees' to them an almost endless life, in spite of their prodigal expenditure of high temperature energy.

Lights of Meteors and Meteor Trains

The cause of the light of meteors is traced to their bombardment by air molecules, when the meteors plunge into our atmosphere with terrific speeds; light being emitted when matter thus evaporating from the body collides with air molecules away from the main body. (It may be pointed out here that repeating shower meteors and most sporadic ones also are found to move in their orbits with velocities somewhat less than-parabolic speeds, viz. 42 kms. per sec. near the earth. They are thus members of the Solar system. Some sporadic meteors, on the other hand, have been observed to move with much higher velocities, signifying motion in hyperbolic orbits, and are therefore considered to be of intersteller origin.)

Thanks to the laboratory experiments of C. C. Trowbridge on nitrogen after-glow, and of Kaplan, Vegard and others on gaseous ionisation, we are beginning to understand the mechanism of the mysterious streaks often seen *near* the end of a meteor's passage through the E region of the ionosphere. Even meteors of such low visibility as those of 2·5 or even 3 m. have been observed by the present writer to display appreciably conspicuous and fairly persistent streaks. Great advancement of our knowledge in this direction will follow if means are devised to obtain *pure* spectra of meteor trains "uncontaminated," so to say, by those of the meteors themselves.

Meteorite Craters

An interesting feature of the study of meteors and meteorites is the subject of meteorite craters, discussed in a number of recent papers. About six such objects have been recognised definitely up till now. The first to attract general attention is that at Canyon Diablo in Arizona, U. S. A., about 4000 ft. in diameter, 570 ft. in depth; the encircling wall rising from

130 to 140 ft. above the level of the surrounding country. Barringer seems to have been the first to suspect its meteoritic origin. Since his time several thousands of pieces of nickeliferous iron, with traces of platinum and iridium (sure signs of meteoritic character) have been picked up, outside the circular boundary of the great depression—one piece as large as 1014 lbs in mass. Search, through drilling, for the buried remnant of the original meteorite (presumably a siderite) that produced the Crater and is suspected to lie beneath its southern rim, has resulted in the recovery of rich meteoritic material from a depth of 1340 ft.

The crater must have been formed by the *explosive* impact of a gigantic meteorite, possibly a small comet or a tiny asteroid that came dangerously near to the earth, in its orbital motion round the Sun, some 10 or 12 thousand years ago. The dread evinced by the Red Indians of the locality in avoiding approach to it, lends support to the notion that a knowledge of the grim phenomenon with its terrible fire-blast and terrific explosion must have been handed down to the present race from generation to generation. Though the angle of hit is reckoned to be about 45° with the horizon, the huge mass must have struck the ground with parabolic velocity, the energy thus released vapourising the bulk of the meteorite and a large amount of the ground hit by it; the recoil of the compressed vapours at once scooping out a round depression and scattering molten matter in all directions.

Craters more or less similar to that in Arizona have been discovered at Wabar in Arabia (two, in 1934), at Odessa, in Ector County, Texas, U. S. A. (one, in 1933), on the island of Ösel in the Baltic, in the Tunguska Valley, Siberia (10 in number), and at Henbury, Central Australia (about 12, in 1931). The Tunguska craters were formed from the impact of large meteorites that fell in that part of Siberia in the early morning of June 30, 1908. Coming at a small angle with the horizon, after impact, they gave rise to a pillar of smoke and dèbris that shot up to a height of 12 miles and was seen from 250 to 300 miles, setting fire to 3000 square miles of forest and causing earthquake shocks that were recorded all over the world!

There is some reason to believe that the Henbury craters were formed at no very remote date in the history of human habitation in Australia, as the name "chindu chinna waru chingi yabu" (Sun walk fire devil road) given by the aborigines of the country to the locality, signifies some acquaintance with the explosive origin of these craters.

It may be pointed out that the danger of a future collision between our Earth and a diminutive asteroid or dwarfish comet is by no means an astronomical fiction, though the devastation likely to be caused by the catastrophe may easily be exaggerated. There have been several dangerously near approaches of such bodies within recent years. Lexell's Comet of 1770 came probably to within 1 or 1½ million miles of the Earth. Adonis (1936 CA) discovered by Delporte came within 1⅓ million miles; and Reinmouth's object discovered in 1937, with a period of only 2·1 years came as close as 4 or 5 hundred thousand miles.

The Earth has probably experienced a number of such hard hits in the course of its 2000 millions of years' existence, and bears only a few dents or

dimples, here and there, from these causes, on its extensive surface. We may therefore, safely assume that the human race need fear no annihilation from such cosmic bombardments. A more serious source of calamity is its love of 'scientific' warfare and aerial raids !

Several geologists, notably Dr. L. J. Spencer, and a few astronomers hold the view that the craters on the moon are also of meteoritic origin. Their great numbers and large areas are not regarded by these investigators as serious obstacles in the formulation of a workable theory.

Concluding Remarks

It will thus be seen that meteoric astronomy is a fascinating new science, with ample opportunities for research, to the inquiring mind. The deflection of well-known meteoric swarms by the perturbing effects of the larger planets, resulting in the anomalous behaviour of meteoric showers, presents a tempting field of investigation to trained mathematicians. As a matter of fact it is a favourite pursuit with a number of well-known observers to calculate the orbits of meteor-swarms that give rise to unusually rich showers and of exceptionally bright fireballs. To experimental physicists, equipped with adequate apparatus and working on the same meteors in accordance with pre-arranged plans at suitable distances, the light of persisting meteor trains is a picturesque problem in atomic physics holding out hopes of a surer knowledge of the condition, character and composition of the inaccessible regions of our atmosphere. The visual observer in his nightly watches discovers newer and newer radiants or confirms old ones, noting every peculiarity of magnitude, colour, apparent speed or duration, concerning all meteors that pass under his survey. The explorer of meteorites unearths and classifies an ever increasing number of these wonderful bodies lying buried underground for perhaps hundreds of years; or saves fresh ones from being lost, through unawareness or neglect. The meteorologist finds in the apparition of meteors new means of unravelling some of the mysteries of the weather lying outside the domain of clouds, fogs and snow. The anthropologist, discovering relics of metallic iron buried with religious care in the tombs of pre-historic men learns, with a little chemical analysis, to fix with greater accuracy the date of discovery of iron-smelting in the past and its bearing on human civilization. More than all this, meteoric astronomy raises the spirit of earth-bound man to cosmic heights, under the open sky and inspires him with veneration for the Great Design revealed in the mechanism of the boundless Universe.

List of those who were present at the 11th session of the Indian Mathematical conference held at Hyderabad from 21st to 23rd Dec. 1939

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